Strengthened Senior High School

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DEPARTMENT OF EDUCATION

Lesson Exemplars for Finite Mathematics 1

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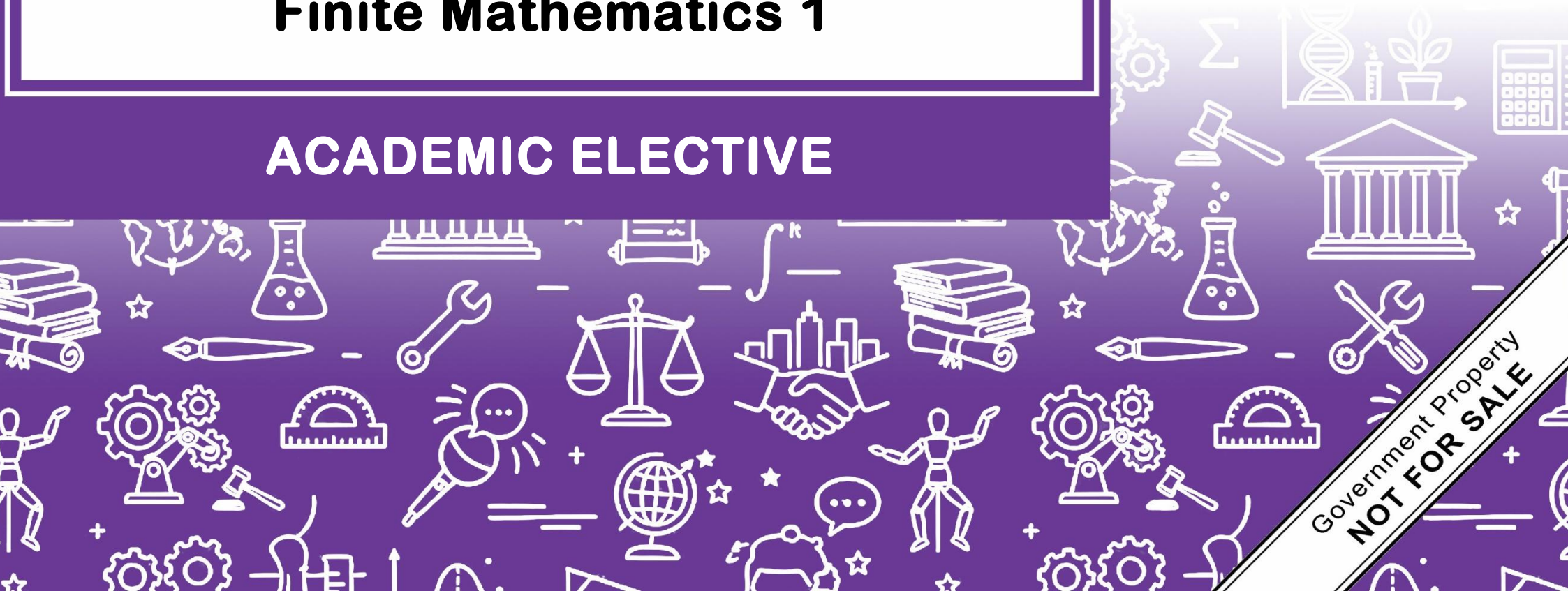

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Lesson Exemplar for Finite Mathematics 1

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Every care has been taken to ensure the accuracy of the information provided in this material. For queries or clarifications, you may contact the Bureau of Learning Resources – Office of the Director at blr.od@deped.gov.ph or via telephone at (02) 8634-1072. If you have feedback on the LE, please accomplish this form, which may be accessed through the following link: **https://bit.ly/sshsfeedbackform**.

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UNIT 1. GEOMETRY OF DESIGN

I. LEARNING GOALS	
Content Standard	The learners demonstrate knowledge and understanding of… 1. Symmetries in nature and art. 2. Different linear transformations (geometric approach). 3. Tiling and Tessellations. 4. Frieze Patterns.
Performance Standard	By the end of the quarter, the learners are able to contribute to a class exhibit of artworks inspired by geometric symmetries, tessellations, Frieze patterns, the golden ratio, the Fibonacci sequence, or fractals. Each artwork should be accompanied by a description detailing the mathematics involved.
Learning Competencies	The learners … 1. identify and describe symmetries in nature and art: a. reflection b. rotational c. line of symmetry d. center of rotation e. angle of rotation 2. illustrate various geometric transformations (translation, reflection, rotation, dilation, and glide reflection); 3. identify polygons that can tessellate the plane; 4. generate tessellations using geometric transformations; 5. identify the geometric transformations that generate a given Frieze pattern.
II. REFERENCES and MATERIALS	
Textbook and Modules	• Department of Education. Strengthened Senior High School Curriculum Guide for Finite Mathematics 1. • Escolastico, J. P., Finite Mathematics 1 Self-Learning Module Quarter 1 Module 1 Symmetry in Nature and Art, DepEd-CAR, 2025 • Reyes, J. A. & Reyes, M. A, (2025). Finite Mathematics 1. Unlimited Books Library Services and Publishing Inc., Manila • Drew, Pippa, and Dorothy Wallace. "Lesson 7: Art and Escher." *Pattern: An Interdisciplinary Course in Textile Design and Elementary Group Theory*. Dartmouth College. Accessed March 12, 2026. https://math.dartmouth.edu/~matc/math5.pattern/lesson7art.html. • Escher, M. C. *The Graphic Work of M. C. Escher*. New York: Ballantine Books, 1967. • Grünbaum, Branko, and G. C. Shephard. *Tilings and Patterns*. New York: W. H. Freeman and Company, 1987. • National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000. • Schattschneider, Doris. *M. C. Escher: Visions of Symmetry*. New York: Harry N. Abrams, 2004.
Websites and Online Articles	• Aniar, Eugene. "Escher Tessellation." *SlideShare*, 2017. https://www.slideshare.net/slideshow/escher-tesselation/81130432.Frieze Examples of Symmetry Symmetry figures in real life • Patterns." *EscherMath*. Accessed March 12, 2026. https://eschermath.org/wiki/Frieze_Patterns.html. • Symmetry in Mathematics Introduction to Symmetry in Mathematics • Filipino Cultural Patterns Filipino cultural pattern references such as traditional textile and design motifs • Symmetry: Radial Symmetry Radial Symmetry in Nature

	• Geometric Transformation Geometric Transformations – Definitions, Types, Examples, and Quiz • Introduction to Geometry-Basic Transformation: MathNation.com • https://www.photopills.com/articles/drone-photography-guide • Symmetry in Mathematics Types of Symmetry • Khan Academy. "Intro to Reflective Symmetry." Introduction to Reflective Symmetry • Khan Academy. "Intro to Rotational Symmetry." Introduction to Rotational Symmetry • Intro to geometric transformations High School Geometry • https://www.khanacademy.org/math/geometry-home/transformations • https://creativecommons.org/licenses/by-sa/3.0 • https://www.scribd.com/document/259135243/Tessellation-in-Architecture
Video lessons	• Geometric Transformations and their rules https://youtu.be/GqHWdTLL8Qw • Khan Academy. (n.d.). *Introduction to Transformations* [Video]. YouTube. https://www.youtube.com/watch?v=XiAoUDfrar0 • Visual Impact Systems – Advertising and Marketing. (n.d.). *M.C. Escher – How to Create a Tessellation* [Video]. YouTube. • https://www.youtube.com/watch?v=ZNVyrxdlrGQ • "Tessellations (Math and Art)." YouTube video. Accessed March 12, 2026. https://youtu.be/ZNVyrxdlrGQ. • "Frieze Patterns and Symmetry." YouTube video. Accessed March 12, 2026. https://youtu.be/7GiKeeWSf4s. • Weisstein, Eric W. "Tessellation." *MathWorld—A Wolfram Web Resource*. Accessed March 16, 2026. https://mathworld.wolfram.com/Tessellation.html. • "Frieze Patterns." YouTube video. Accessed March 12, 2026. https://youtu.be/524gLKdaMZM
Instructional Materials	• Learning Activity Sheets and worksheets • printed and visual learning materials (picture of different object) • writing and drawing materials • manipulatives and hands-on materials
EdTech	• GeoGebra app and drawing applications • WAYGROUND Formerly Quizizz Geometry-Transformation Pre-assessment
Use of Generative AI	In preparing this Learning Exemplar (LE), the author(s) used generative artificial intelligence (AI) tools to assist in organizing ideas, developing sample activities, and generating formative assessment items. The author(s) carefully reviewed, validated, and finalized all content. The author(s) take full responsibility for the content of this LE.

III. CONTENT	**Lesson 1.1: SYMMETRIES IN NATURE AND ARTS**	
IV. OBJECTIVES	**At the end of the lesson, the learners will able to:** **1. identify and describe symmetries in nature and art;** **2. identify different types of symmetry such as reflection and rotational symmetry;** **3. determine the center of rotation, angle of rotation, and order of symmetry using cut-outs or GeoGebra; and** **4. describe the rotational symmetry of objects found in nature and Filipino design.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)** *(Description and Purpose of Each Activity, Strategies, Summative Test and Performance Task, and Answer Keys Refer to Teachers Note)*

<table>
<tr><td rowspan="4">A. Activating Prior Knowledge</td><td colspan="2">A.1. Eliciting Prior Knowledge</td></tr>
<tr><td>Activity A.1
LEVELING LEARNERS READINESS: DISCOVERING SYMMETRY AROUND US THROUGH GALLERY WALK

Instructions:
1. You will work in groups of 4–5 members to complete an activity on identifying reflection and rotational symmetry.
2. Around the classroom are pictures and objects inspired by nature and Filipino culture, such as butterflies, flowers, shells, leaves, woven banig, parol designs, and textile patterns.
3. As a group, move around the room and carefully observe each image or object.
4. Select three objects or designs that appear balanced, repeated, or patterned.
5. Discuss the following questions with your group:
a. What makes the object or design look balanced or organized?
b. What repeated shapes, lines, or patterns do you notice?
c. Do you think the object shows symmetry? Explain your answer.
6. Classify each chosen object as showing reflection symmetry, rotational symmetry, or both.
7. Record your observations on the activity sheet using simple mathematical terms such as symmetry, line of symmetry, reflection, and rotation.
8. Each group will present one example to the class and explain why the object or design demonstrates reflection or rotational symmetry.

Processing Questions:
1. Which objects or designs appear to have reflection symmetry?
2. Which objects might have rotational symmetry?
3. Why do you think symmetrical patterns are common in nature and arts?</td><td>Activity A.1
LEVELING LEARNERS READINESS: DISCOVERING SYMMETRY AROUND US THROUGH GALLERY WALK

1. Prepare and display pictures of objects from nature and Filipino designs around the classroom.
2. Organize the learners into groups of 4–5 members.
3. Explain that learners will observe the images and identify patterns, balance, or repetition that may indicate symmetry.
4. While learners move around the room, ask these guiding questions:
a. What patterns do you notice in this object?
b. Do the two sides of the object look the same?
c. Does the design appear balanced or repeated?
5. Prompt learners to use mathematical terms such as symmetry, line of symmetry, and reflection when describing their observations.
6. Walk around the classroom to ensure that all groups are engaged, provide clarification when needed, and encourage quieter learners to participate.
7. After the gallery walk, ask each group to share one object they observed and explain why they think it shows symmetry.
8. Summarize the learners' observations and explain that these patterns relate to the mathematical concept of symmetry, which will be explored further in the lesson.</td></tr>
<tr><td colspan="2">A.2. Establishing the Purpose of the Lesson</td></tr>
<tr><td>Activity A. 2
APPRECIATING LESSON RELEVANCE: WHY STUDY SYMMETRY
Instructions:</td><td>Activity A. 2
APPRECIATING LESSON RELEVANCE: WHY STUDY SYMMETRY</td></tr>
</table>

	1. Look again at the examples you observed during the Gallery Walk. 2. With your group, imagine you are designers creating a new textile pattern or object for a festival. 3. Discuss the following: a. How can symmetry help make your design balanced and attractive? b. Why do many natural objects such as flowers or butterflies appear symmetrical? 4. Observe and analyze the additional examples of Filipino architecture, patterns, and natural objects presented by your teacher. Identify patterns and symmetry in each example. 5. Based on your observations, identify why symmetry might be important in real life. 6. Share your group's ideas with the class. **Processing Questions:** 1. How does symmetry help artists, architects, and designers in the Philippines create visually pleasing designs? 2. How might symmetry help us understand patterns in nature and the environment? 3. Why do you think mathematicians study symmetry?	1. Display again several examples from the gallery walk, such as butterflies, flowers, shells, or Filipino textile patterns. 2. Ask learners guiding questions such as: a. Why do these objects look balanced or attractive? b. What do these patterns have in common? 3. Allow learners to discuss their ideas with a partner or small group. 4. Encourage learners to relate symmetry to real-life applications, such as architecture, art, product design, and natural structures. 5. Summarize the learners' responses and explain that the lesson will focus on identifying and describing symmetry in nature and art. 6. To end the activity and introduce the lesson, the objectives and the performance task to be performed.
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. **EXPLORING KEY CONCEPTS:** **FOLD-TEST AND MIRROR ACTIVITY** **Instructions:** 1. Stay with your group of 4–5 members. 2. Each group will receive printed images of natural objects and Filipino patterns, such as butterflies, leaves, shells, flowers, and woven designs. 3. Examine each image and predict whether the figure has symmetry. 4. Perform the Fold-Test: a. Gently fold the paper along a possible line. b. Check if the two halves match exactly. 5. Perform the Mirror Test: a. Place a small mirror along the line you identified.	**Activity B.1.** **EXPLORING KEY CONCEPTS:** **FOLD-TEST AND MIRROR ACTIVITY** 1. Organize the class into groups of 4–5 learners and distribute the printed images, mirrors, activity sheets, and drawing materials before the activity begins. 2. Review the meaning of reflection symmetry and demonstrate how to perform the fold-test and mirror test using one sample image. Emphasize careful handling of materials and proper use of mirrors. 3. Encourage learners to discuss their predictions collaboratively before testing each object. Prompt them to explain their reasoning using terms such as "equal halves," "mirror image," and "line of symmetry." 4. While groups are working, circulate around the classroom to observe participation, ask guiding questions, and provide

b. If the reflection completes the figure, the object shows reflection symmetry.
6. Use a marker or pencil to draw the line(s) of symmetry.
7. Record your observations in the Symmetry Investigation Table:

Object	Prediction	Has Reflection Symmetry?	Number of Lines of Symmetry

8. Each group will present one object and explain how they confirmed its symmetry.
9. Optional Technology Extension: Use GeoGebra to digitally reflect the image and verify the symmetry.

support to learners who may have difficulty identifying lines of symmetry.
5. Remind learners to record accurate observations in the Symmetry Investigation Table and ensure that each member contributes to the discussion and testing process.
6. During the presentation phase, encourage groups to clearly explain how they confirmed symmetry using the fold-test or mirror test. Allow other learners to ask questions or share additional observations.
7. Synthesize the activity by connecting learners' findings to real-life examples of symmetry found in nature, Filipino arts, architecture, weaving patterns, and design. Highlight the importance of symmetry in beauty, balance, and functionality.
8. If technology is available, guide learners in using GeoGebra to digitally reflect images and verify their answers.

Processing Questions:
1. How did the fold-test or mirror test help you confirm reflection symmetry?
2. Can a shape have more than one line of symmetry? A square is one example. Can you give more examples?
3. Which objects from nature or Filipino designs showed reflection symmetry?
4. Why do designers and artists often use symmetry in their work?

B.2. Discussing the Concept

Activity B.2.
DEEPENING CONCEPTUAL UNDERSTANDING: EXPLORING SYMMETRY IN NATURE AND IN ARTS

Instructions:
Study the key concepts and examples with your teacher's guidance and your classmates' support. Work together to understand and explain the importance of the different types of symmetry in nature and art.

Part I. Key Concepts and Examples
Have you ever noticed how a butterfly's wings look the same on both sides? Or how a snowflake has beautiful repeating patterns?

Activity B.2.
DEEPENING CONCEPTUAL UNDERSTANDING: EXPLORING SYMMETRY IN NATURE AND IN ARTS

1. Prepare key examples: Display images or slides of butterflies, flowers, starfish, snowflakes, parol designs, Filipino textile patterns (inabel, banig), and architectural elements.
2. Guide learners through observation and reading: Ask learners to read and analyze the examples provided. Highlight connections between nature, art, and mathematical concepts.
3. Ask probing questions: Encourage learners to think critically:
   a. What patterns do you notice?
   b. Why do you think nature or artists use symmetry?

	What about the balanced beauty of a building or the repeating patterns in a piece of artwork? That's symmetry—a concept found all around us in both the natural world and human-made art. **Symmetry in Nature** The natural world is full of beauty and amazing shapes and patterns. Ian Stewart, in his book *Nature's Numbers* (1995), describes mathematics as the science of patterns and shows how the patterns observed in nature can be expressed using mathematical language. In nature, symmetry is often a sign of balance, health, or function. Many plants and animals have bilateral symmetry, meaning one side is a mirror image of the other. It is an exact correspondence of form and constituent configuration on opposite sides of a dividing line or plane or about a center or an axis. Other natural objects show radial symmetry a type of symmetry where the parts of an object are arranged around a central point—like the spokes of a bicycle wheel or slices of a pizza. Nature uses symmetry for many reasons: 1. To attract mates (like the patterns on peacocks) 2. For balance and movement (as in animals) 3. To grow in a stable and efficient way (like leaves and shells) **Symmetry in Art** Artists and architects have used symmetry for thousands of years to create beauty, harmony, and order. Whether it's the intricate patterns in Islamic tiles, the balance in classical architecture, or the radial designs in mandalas, symmetry helps art feel pleasing and structured. In art, symmetry can be: **Reflective (Mirror) Symmetry** – occurs when one half of an object or shape is a mirror image of the other half. If you can draw a line down the middle (called the line of symmetry) and both sides match exactly, then the shape has reflection symmetry. It's like folding a picture in half and having both sides line up perfectly!	c. How can we classify the symmetry: reflection symmetry, rotational symmetry, both, or neither? 4. Facilitate discussion: Encourage learners to share their observations with peers and discuss examples from both nature and Filipino designs. 5. Summarize key ideas: Emphasize definitions and applications of symmetry, center of rotation, angle of rotation, and order of symmetry in both natural and artistic contexts. To understand the rotational symmetry, facilitate any of the two activities. **Option I.** 1. Organize learners into groups of 3–4 and distribute cut-outs of starfish, flowers, parol designs, circular textile patterns. 2. Guide learners to identify the center of rotation and rotate the object slowly in increments, observing when it matches the original position. 3. Ask learners to record the center of rotation, angle of rotation, and order of symmetry in their tables. 4. Facilitate group discussions: a. Why do some objects show rotational symmetry and others do not? b. Can a design have both reflection and rotational symmetry? 5. Invite groups to present one example and explain their observations. **Option II.** 1. Guide learners to use GeoGebra or any rotation tool. 2. Import images (starfish, flowers, parol patterns) and identify the center of rotation digitally. 3. Rotate the objects in increments and record angle of rotation and order of symmetry. 4. Encourage learners to experiment with multiple objects and compare rotational symmetry. 5. Facilitate pair discussions on similarities, differences, and patterns observed.

Rotational Symmetry – designs that repeat around a central point

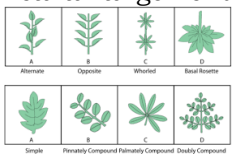
Translational Symmetry - patterns that repeat across space. This happens when a shape, pattern, or design can be slid in certain direction and still look exactly the same.
The shape doesn't rotate or flip, it moves a certain distance in straight line and repeats itself.

Example:

Fabric.

Brick wall.

Floor Tile

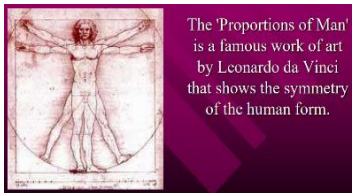
Leaf arrangement 

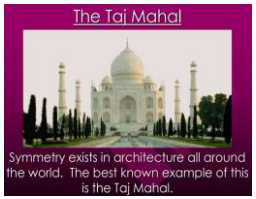
Key Parts of Rotational Symmetry:
Center of rotation – the spot the shape rotates around.
Angle of rotation – how far you turn the shape each time it looks the same.
Order of symmetry – how many times the shape matches itself in one full turn.

Some patterns repeat by sliding rather than by reflecting or rotating. This is called translational symmetry, and you will study it when you take up linear transformations and tessellations and Frieze patterns other lesson

The importance of symmetries in nature and art in terms of:
1. Reflection Symmetry (Mirror Symmetry)
Nature: Many living organisms exhibit mirror symmetry. For example, butterflies and human bodies are bilaterally symmetrical—one side is a mirror image of the other. This symmetry is often linked to biological function and evolutionary advantages like streamlined movement or balanced sensory input.

Art: In visual art and architecture, mirror symmetry is used to create harmony and a sense of order. Traditional Islamic art, for instance, heavily relies on reflection symmetry to produce visually pleasing patterns.





2. Rotational Symmetry

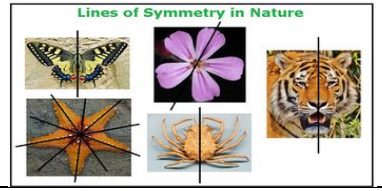
Nature: Flowers, starfish, and snowflakes often show rotational symmetry. For instance, a five-petaled flower has fivefold rotational symmetry. This type of symmetry helps in maximizing exposure to sunlight or efficient reproduction.

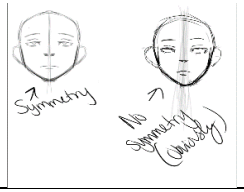
Arts: Mandalas and circular designs use rotational symmetry to evoke balance and spirituality. Artists use it to guide the viewer's eye around a central point, creating unity in the composition.

3. Line of Symmetry

Nature: A line of symmetry divides an object into two identical parts. Leaves, insects, and many animals possess a vertical line of symmetry. It often reflects functional optimization, like flight in birds or balance in humans.

Arts: Lines of symmetry are used to establish focal points or create balanced compositions. A portrait may use a vertical line of symmetry for a formal and composed look.





4. Center of Rotation

***Nature*:** The center of a sunflower or a jellyfish is a clear example of where rotational symmetry originates. It helps distribute resources evenly (like seeds in a sunflower).

***Art*:** In decorative art, like rose windows in Gothic cathedrals, the center of rotation is often the visual focus, radiating symmetry outward to engage the viewer.

5. Angle of Rotation

Nature: The angle of rotation defines how many times an object maps onto itself in a full 360° turn. For example, a snowflake typically has 60° rotational symmetry (sixfold).

Art: In pattern design or tessellations, the angle of rotation defines how elements repeat and interlock, essential for creating intricate and mesmerizing visuals.

Rotational Symmetry in Nature And Filipino Designs

Before performing the activity, the teacher should state the following: For a figure with rotational symmetry of order n, the smallest angle of rotation is computed as 360 degrees divided by n. For an idealized 5-arm starfish, n = 5, so the smallest angle is 360 / 5 = 72 degrees. For an idealized 8-point parol, n = 8, so the smallest angle is 360 / 8 = 45 degrees.

Practice Items (Warm-up before the main activity):

1. A regular hexagonal flower has rotational symmetry of order 6. Compute the smallest angle of rotation. Verify your answer by rotating a paper cut-out or a GeoGebra image and checking that the figure matches itself after each turn.
2. A circular Filipino textile motif has rotational symmetry of order 3. Compute the smallest angle of rotation. Then sketch or model

	the motif and verify that turning it through that angle produces the same figure. **Option 1. Using cut-outs (Hands-on)** **Instructions**: 1. Form a group of 3–4 members. 2. Each group receives cut-outs of objects such as: a. Starfish b. Flowers with multiple petals c. Parol designs d. Circular Filipino textile patterns 3. Place the cut-out on a flat surface and identify the center of rotation (the point around which the shape can rotate). 4. Slowly rotate the cut-out around the center in increments. 5. Observe the points at which the shape matches its original position. 6. Record in your table: a. Center of Rotation b. Angle of Rotation (how far you rotate to match the shape) c. Order of Symmetry (number of times it matches in one full 360° turn) 7. Discuss in your group why some objects show rotational symmetry and others do not. 8. Each group presents one example and explains your observations. **Option 2: Using GeoGebra (Technology-Enhanced)** **Instructions**: 1. Open GeoGebra (or any rotation tool on the computer or tablet). 2. Choose or import an image of an object such as: Starfish, Flower, and Parol design. 3. Select the rotation tool in GeoGebra. 4. Identify the center of rotation on the screen. 5. Rotate the object in increments and observe when it matches the original position. 6. Record in your table: a. Center of Rotation b. Angle of Rotation c. Order of Symmetry 7. Use GeoGebra to experiment with multiple objects and compare rotational symmetry across different shapes.	

<table>
<tr><td></td><td colspan="2">8. Discuss in pairs why some objects exhibit rotational symmetry while others do not.

Processing Questions:
1. How do you determine the center of rotation?
2. What is the angle of rotation for the starfish or parol?
3. Can an object have both reflection and rotational symmetry? Give examples.
4. How does understanding rotational symmetry help in designing or analyzing patterns?</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3.
PRACTICING LEARNED SKILLS: SYMMETRY ANALYSIS TABLE

Before facilitating the activity, please have the learners bring out their Learning Activity Sheets (LAS).
1. Prepare materials: Provide each group with printed objects or images such as butterflies, starfish, flowers, parol designs, and circular Filipino textile patterns. Ensure learners have access to the Symmetry Analysis Table.
2. Introduce the activity: Explain that learners will apply the knowledge they gained from previous activities to classify objects based on their reflection and rotational symmetry. Highlight that they should identify line(s) of symmetry, center of rotation, and angle of rotation for each object.
3. Guide learners during table completion: Circulate around the room to observe, prompt, and support learners. Ask guiding questions like:
a. Does folding this object along a line create matching halves?
b. Where is the center of rotation for this object?
c. How far does the object rotate to match itself again?
4. Facilitate pair-share discussion: Encourage learners to compare answers with a partner, justify their reasoning using mathematical language, and discuss differences in observations.
5. Review and clarify: After pair-share, lead a class discussion to address misconceptions. Highlight key points: for example, some objects have only reflection symmetry, others only rotational symmetry, and some have both. Emphasize why multiple lines of symmetry affect a figure's appearance.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1
MAKING REAL-WORLD CONNECTION: SYMMETRY INVESTIGATION

Instructions:
1. Group Organization: Form groups of 3–4 learners. Each group will examine a variety of local Filipino designs (textiles, parol patterns, church windows), natural objects (leaves, flowers, shells, starfish), and geometric shapes.</td><td>Activity C.1
MAKING REAL-WORLD CONNECTION: SYMMETRY INVESTIGATION

1. Prepare a selection of local Filipino designs, natural objects, and geometric shapes for each group.
2. Explain the activity clearly: learners will observe objects, identify symmetry types, and record properties (line of</td></tr>
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2. Observation and Analysis: Carefully observe each object or design. Identify the type of symmetry (reflection symmetry, rotational symmetry, both, or neither) and note important properties.
3. Complete the Symmetry Table: Record your observations using the table below:

Object	Type of Symmetry	Line of Symmetry	Center of Rotation	Angle of Rotation

4. Group Discussion and Justification:
   a. Discuss with your group why you classified each object as having a particular symmetry type.
   b. Justify your answers using mathematical language: line of symmetry, center of rotation, angle of rotation.
   c. Identify any patterns or trends among Filipino art, nature, and geometric shapes.
5. Presentation:
   a. Each group presents one or two examples to the class.
   b. Explain the type of symmetry, properties, and why it matters in real-life design or observation.

Processing Questions:
1. Which symmetry types are most common in Filipino art?
2. How does rotational symmetry appear in nature?
3. Why is symmetry important in design and observation?

symmetry, center of rotation, angle of rotation). Emphasize the use of mathematical terms when justifying observations.
3. Guide learners as they complete the Symmetry Table, circulating around the classroom to ask probing questions, check understanding, and help groups analyze patterns across objects.
4. Facilitate group presentations: encourage learners to share examples, explain reasoning, and connect symmetry concepts to real-life applications in Filipino art, nature, and design.
5. Clarify misconceptions during presentations and highlight patterns or trends, such as the frequent use of reflection symmetry in Filipino textiles and rotational symmetry in flowers or circular designs.

C.2. Making Generalization

Activity C.2.
WRAPPING UP THE LESSON

Option 1. SYMMETRY CONCEPT MAP
Instructions:
1. In pairs, create a concept map about Symmetry.
2. Include the following concepts:
   a. Reflection Symmetry
   b. Rotational Symmetry
   c. Line of Symmetry

Activity C.2.
WRAPPING UP THE LESSON

Option 1. SYMMETRY CONCEPT MAP
1. Learners work in pairs and are given chart paper or digital tools to create a concept map about symmetry.
2. Learners organize and connect key ideas such as reflection symmetry, rotational symmetry, line of symmetry, center of rotation, and angle of rotation.

	d. Center of Rotation e. Angle of Rotation 3. Add examples from nature, Filipino art, and geometric shapes. 4. Connect the concepts using arrows and short explanations. **Option 2. SYMMETRY DETECTIVE** **Instructions:** a. Look around the classroom or your surroundings. b. Identify 3 objects that show symmetry. c. Write: a. Type of symmetry b. Line of symmetry or center of rotation c. Why the design might use symmetry. d. Generalize from your observations: Across the classroom objects you found, what general statements can you make about where reflection symmetry tends to appear and where rotational symmetry tends to appear? State two patterns you observed and justify each with one example.	3. Learners add examples from nature, Filipino art, and geometric shapes, and use arrows or labels to show how the concepts are related. 4. Teacher moves around the classroom to ask guiding questions and support learners in making accurate connections between ideas. 5. Select pairs to present their concept maps to the class, explaining how the concepts are related. 6. The teacher then facilitates a short synthesis discussion to reinforce the relationships among symmetry concepts. **Option 2. SYMMETRY DETECTIVE** 1. Learners observe objects within the classroom or their surroundings that show symmetry. 2. They record at least three objects found during their observation. 3. They identify the type of symmetry (reflection or rotational) for each object. 4. They determine the line of symmetry or center of rotation when applicable. 5. They explain why symmetry may be used in the design of each object. 6. Learners may work individually or with a partner during the activity. 7. The teacher facilitates learning by asking guiding questions that deepen understanding of symmetry in real-life contexts. 8. Learners share their findings with the class after the investigation. 9. The class discusses how symmetry is present in everyday objects and structures
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS)	
	Activity C.3 **ASSESSING LEARNING OUTCOMES** Before facilitating this activity, ensure that learners have a copy of the LAS. 1. Instruct them to carefully observe each image on the LAS you may also present these images on the board. a. Type of symmetry b. Location of symmetry (line or center) c. Justification of their answers 2. Encourage learners to explain *why*, not just *what*.	

3. Process the guide questions.
    a. Learners should point out vertical, horizontal, or diagonal lines dividing the figure equally.
    b. Learners identify the fixed point (usually the center).
    c. Answers may include 90^0, 120^0, 180^0, etc., depending on the figure.
    d. Learners justify using properties of equal division and repeated rotation.
    e. Accept varied answers; evaluate based on reasoning (e.g., circle = infinite symmetry).
4. Let learners compare answers in pairs. Encourage them to defend their reasoning using mathematical terms.
5. Facilitate sharing of answers. Address misconceptions such as:
    a. Misconception 1: Confusing rotation with reflection. Diagnostic: ask, "Can you fold this image in half so the two parts match?" If yes, the image has reflection symmetry. Now turn the image around its center. Does it match itself before a full turn? If yes, it also has rotational symmetry. Remediation: have the learner use a small mirror to check reflection and a paper cut-out to physically rotate.
    b. Misconception 2: Treating 360 degrees as a non-trivial rotation. Diagnostic: ask, "Does turning the figure all the way around count as a new position?" Remediation: state the convention that the smallest non-zero angle of rotation is what we name as the angle of rotation, and the order is how many distinct positions the figure passes through in one full turn including the starting position.
    c. Emphasize correct concepts using learner responses.

C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS)

Activity C.4
EXTENDING AND REINFORCING LEARNINGS

Before facilitating this activity, ensure that learners have a copy of the LAS.

Option 1. CREATE YOUR OWN SYMMETRICAL ART

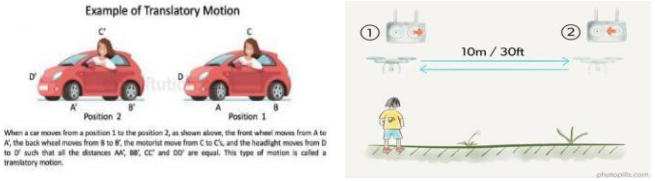
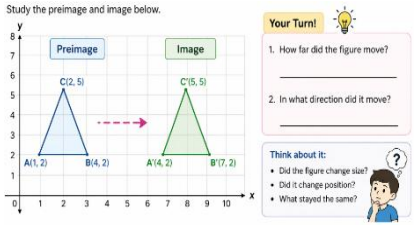
1. Explain to learners that they will create their own symmetrical designs, either on paper with colored pencils/markers or digitally using GeoGebra or a drawing app.
2. Encourage learners to draw inspiration from nature, Filipino cultural patterns, or geometric shapes.
3. Guide learners to identify and label symmetry properties:
    a. Reflection symmetry: draw the line(s) of symmetry.
    b. Rotational symmetry: mark the center of rotation, calculate the angle of rotation, and note the order of symmetry.
4. Monitor learners' progress, asking guiding questions such as:
    a. How did you determine your line of symmetry or center of rotation?
    b. How does your design show balance, repetition, or harmony?
5. Facilitate presentations where each learner explains their design and symmetry properties to the class or small groups.
6. Encourage peer feedback: classmates may ask questions, provide suggestions, or comment on the artistic and mathematical aspects of the design.

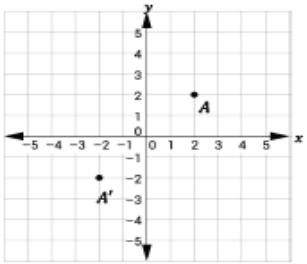
Option 2. GALLERY WALK REFLECTION

1. Organize a Gallery Walk, where learners circulate around the classroom to observe and analyze their classmates' symmetrical designs.
2. Provide feedback sheets for learners to write short comments or questions:

	3. Identify the type of symmetry used. a. Note positive observations about the design. b. Ask a question or suggest improvements. c. After the walk, discuss their observations in pairs. 4. Facilitate a whole-class discussion highlighting interesting symmetry patterns, cultural connections, and artistic techniques.

III. CONTENT	**Lesson 1.2: TRANSLATION**	
IV. OBJECTIVES	**At the end of the lesson, the learners are will be able to:** **1. define geometric transformation and identify its types;** **2. define translation as a movement that slides a figure without changing its size and shape;** **3. describe translation in terms of direction and distance on the coordinate plane;** **4. perform translation of points and simple geometric figures using given rules; and** **5. illustrate translation of figures on the coordinate plane accurately.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNERS READINESS **Instruction**: Do the indicated tasks **Part I. Translation Observation in the Real World** 1. Work in pairs. 2. Look around the classroom or outside the window for objects that appear to have moved without changing shape or size. 3. Discuss with your partner how these objects seem to have "slid" or changed position. 4. Share one example with the class and describe it in your own words. **Part II. Sketching Preimage–Image Pairs** 1. Use the provided grid paper with a simple shape (triangle, square, or rectangle). 2. Draw the given shape as your preimage. 3. Move the shape to a new location on the grid without changing its size or orientation. 4. Draw the new position as the image. 5. Label the original shape as preimage and the new shape as image. **Part III. GeoGebra Exploration** 1. Open GeoGebra or another available digital geometry tool.	**Activity A.1. LEVELING LEARNERS READINESS** 1. Organize learners into pairs and instruct them to observe real-life examples of sliding movements inside or outside the classroom, such as tiled floors, aligned chairs, or repeated wall patterns. 2. For technology integration, demonstrate how to use GeoGebra translation tools and allow learners to explore the movement of figures digitally. 3. Ask guiding questions and encourage learners to describe their observations using their own words. **Processing Questions**: 1. What stayed the same about the shape after moving it? 2. How did its position change? 3. Can you describe the movement in words?

	2. Select a shape on the workspace. 3. Use the translation tool to slide the shape across the coordinate plane. 4. Observe what happens to the position of the figure while its size and shape remain unchanged.	
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. APPRECIATING LESSON RELEVANCE **Option 1. Why Study Translation** **Instructions:** 1. Observe the real-life examples presented, such as a car moving along a street, a drone flying across a grid, and tiles in a floor pattern.  2. Think about and discuss: Why is it useful to understand how objects move without rotating or changing size? **Option 2. Predicting Translation Rules** Instructions: 1. Analyze the pictures below.  2. Guess how far and in what direction the figure moved, discuss with your classmates. 3. This lesson will help you describe and perform such movements mathematically using rules. 4. Answer the question: "How do architects, graphic designers, or engineers use sliding movements of shapes in their work?"	**Activity A.2. APPRECIATING LESSON RELEVANCE** **Option 1.** 1. Present images (car, drone, tiles). Ask guiding questions: a. What do you notice about the object as it moves? b. Did its size or shape change? 2. Encourage learners to answer before confirming. **Option 2.** 1. Show a preimage and image on a coordinate plane. 2. Ask where did the figure move? How many units to the right/left/up/down? 3. Let learners estimate first before confirming exact answers. 4. Let learners think individually first (30–60 seconds). 5. Pair them to discuss answers. 6. Call selected pairs to share with the class. 7. Affirm correct ideas and clarify misconceptions. After completing the Think-Pair-Share discussion, introduce the unit performance task.

B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Part I. Teacher Demonstration** Instructions: 1. Answer the following questions: a. What do you think happens when you transform a figure? b. What are some different ways that you can transform a figure? c. Fill in the blanks. ● In geometry, transformations refer to the _____ of objects on a coordinate plane. ● A _____ is the original object. ● The _____ (') is used to represent a transformed figure of the original figure. 2. Consider the graph below, circle the pre-image and box the transformed image. Describe the transformation.  3. There are two main categories of transformations: rigid and non-rigid. a. A _____ transformation changes the size a. of the pre-image. b. A _____ transformation does not change a. the size of the pre-image. 4. Write a real-world example of a rigid transformation. e. Write a real-world example of a non-rigid transformation. There are five common types of transformations covered in this unit: a translation slides the shape; a reflection flips the shape across a line; a rotation turns the shape around a center point; a dilation changes the size of the shape (scale factor); and a glide	**Activity B.1. EXPLORING KEY CONCEPTS** **Part I.** 1. Begin by asking the guide questions to activate prior knowledge. 2. Allow learners to share ideas before giving formal definitions. 3. Present a sample graph and guide learners to identify the pre-image and image (transformed figure). 4. Model how to circle and box correctly. 5. Introduce rigid and non-rigid transformations using simple examples. **Part II** 1. Given the type of transformation (translation, reflection, rotation, or dilation), determine if it is rigid or non-rigid. Provide a short justification. 2. Identify the transformations shown in the graph. a. Translation- The figure is moved to a different location without changing its shape, size, or orientation. b. Rotation- The figure is turned around a point (center), changing its orientation but not its size or shape. c. Reflection- The figure is flipped across a line (mirror line), creating a mirror image. d. Translation- The figure slides horizontally to the right with no change in size or orientation. **Part III**. 1. Demonstrate briefly how to: a. Plot points in GeoGebra b. Use the Translate by Vector tool OR manually apply the rule 2. Provide the given example: $A(1,2) \rightarrow A'(4, 4)$ $B(3, 2) \rightarrow B'(6, 4)$ $C(2, 4) \rightarrow C'(5, 6)$ 3. Guide learners step-by-step, especially those unfamiliar with the tool. 4. Ask guiding questions during exploration: a. What happened to each point? b. Did the size change? c. Did the figure rotate or flip?

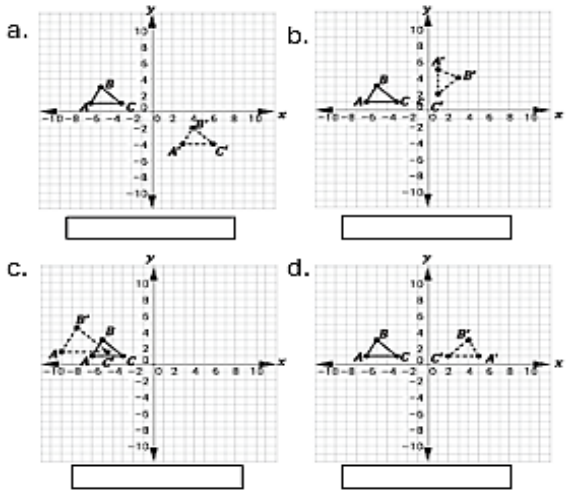
reflection combines a reflection with a translation parallel to the line of reflection. Today we focus on translation. The other four are covered in LE 1.3 to LE 1.7.

Part II. Learner Identifying
Instructions:

1. In the table, indicate whether the transformation is rigid or non-rigid and justify your answer.

Transformation	Rigid (R) or Non Rigid (NR)	Justification
Translation		
Reflection		
Rotation		
Dilation		

2. Now, identify the transformations shown in the following graphs and write the names of the transformations in the corresponding boxes under each graph.

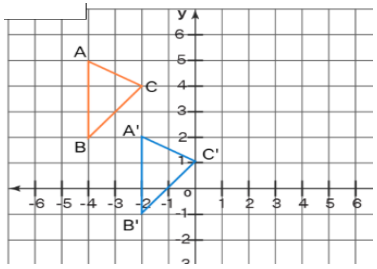


Part III. GeoGebra Exploration
Instructions:

1. Open GeoGebra and plot the triangle with vertices:
   A(1, 2), B(3, 2), C(2, 4)

5. Allow learners to explore independently after the guided example.

Non-technology option: Use graph paper. Plot the preimage, apply the coordinate rule to each vertex, and draw the image. Compare the image to the preimage by measuring corresponding sides and angles.

	2. Apply the translation rule: $(x, y) \rightarrow (x + 3, y + 2)$ 3. Plot the new image A', B', C' after translation. 4. Compare the pre-image and image then identify what changed and what stayed the same. 5. Drag the original figure and observe what happens to the image.	
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING P**art I. Understanding Translation Through Words and Observation** 1. Read and study the description of translation and how figures move on the coordinate plane. A **transformation** is a rule that maps each point of a figure, called the preimage, to a corresponding point of its image. A translation is a transformation that moves every point the same distance and direction. It does not turn, flip, stretch, or shrink the figure. Because every point moves by the same vector, a translation preserves length, angle measure, shape, size, and orientation. On the coordinate plane, a translation can be written as (x, y) → (x + a, y + b), where a tells how far the point moves horizontally and b tells how far it moves vertically. 2. Observe the given example of triangle ABC translated to triangle A′B′C′. Verify how each point moved by checking the horizontal and vertical changes in the coordinates.  3. Discuss the following questions with your partner or group: a. How far did each point move? b. What stayed the same after the translation? c. Why is translation described as a “slide”? 4. Describe the movement of the triangle in words, such as “move 3 units right and 2 units up.”	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** Introduce the term translation and explain the coordinate rule: (x, y) → (x + a, y + b) **Part I**. 1. Present both pre-image and image on the grid. 2. Guide learners to compare A(-4, 4) → A'(-2, 2) 3. Ask what happened to x-values and to y-values. 4. Lead learners to describe movement in words before introducing rules. **Part II.** 1. Introduce the general rule: (x, y) → (x + a, y + b) 2. Demonstrate one example step-by-step: A(1, 2) → A'(3, -1) 3. Emphasize: a. Add to x for horizontal movement b. Add to y for vertical movement **Part III.** 1. Present the puzzle as a challenge. 2. Ask what is the rule for ‘3 units left and 2 units up’. 3. Guide learners to write: (x, y) → (x - 3, y + 2) 4. Let learners solve individually, then discuss as a class.

	5. Share your description with classmates and compare your observations. **Part II. Applying Translation Rules on the Coordinate Plane** 1. Study the translation rule given by your teacher. 2. Apply the rule to the points A(1,2), B(3,4), and C(-1,5). 3. Write the new coordinates A′, B′, and C′. 4. Plot both the preimage and image on the coordinate grid. 5. Compare the two figures and identify what changed and what remained the same. 6. Explain how the translation rule helped you determine the new coordinates. **Part III. Translation Challenge Puzzle** 1. Study the given preimage points A(2,1), B(4,1), and C(3,3). 2. Apply the translation "move 3 units left and 2 units up." 3. Predict the image coordinates A′, B′, and C′ before plotting them on the grid. 4. Explain how you determined your answers. 5. Plot the preimage and image to check your predictions. 6. Reflect on how translation can describe movement in mathematics and real-life situation.	
	B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)	
	Activity B.3. PRACTICING LEARNED SKILLS Before facilitating this activity, ensure that all learners have a copy of the LAS. **Facilitation Guide:** 1. Direct learners to apply the translation rule (x, y) → (x − 3, y + 2) to triangle ABC with A(2, 1), B(5, 1), C(2, 4). 2. Guide learners to verify that each x-coordinate decreases by 3 and each y-coordinate increase by 2. 3. Ask learners to measure AB and A′B′. Guide them to confirm that length is preserved. Length- count horizontal units Midpoint- average of x and y 4. Present only the translation rule (x, y) → (x − 3, y + 2). 5. Apply it to A(2, 1), B(5, 1), C(2, 4) to obtain A′(−1, 3), B′(2, 3), C′(−1, 6). 6. Demonstrate that AB = A′B′ = 3 units. Confirm that only the position changed. 7. Model one step, then let learners proceed independently.	
C. Demonstrating Knowledge and Skills	***C.1. Finding Practical Application***	
	Activity C.1 **MAKING REAL-WORLD CONNECTIONS: MINI-PRACTICE FOR THE TRANSFORMATION SPECIALIST ROLE**	**Activity C.1** **MAKING REAL-WORLD CONNECTIONS: MINI-PRACTICE FOR THE TRANSFORMATION SPECIALIST ROLE**

	Instructions: 1. Choose one base figure (a triangle, square, or pentagon). 2. Show three translations of the figure with vectors (3, 0), (0, 4), and (−2, −3) 3. Plot the pre-image and the three images on a coordinate grid. Label each image with the translation vector used. 4. Write a one-paragraph justification (75 to 100 words) explaining which property of translation guarantees that the four figures (pre-image and three images) are congruent.	The teacher presents images of architectural layouts, floor tiles, and logo patterns. Learners identify translations and create coordinate treasure maps requiring translation rules to reach targets.
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON 1. Discuss as a class: a. Translation slides a figure without changing size or shape. b. It is described by direction and distance on a coordinate plane. c. Coordinate rule: (x, y) → (x + a, y + b) 2. Write your own summary in words and coordinates.	**Activity C.2. WRAPPING UP THE LESSON** To leads synthesis discussion, activity C.2 is provided. Learners write summaries describing translation in words and symbols. .
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Instruct learners to answer individually on a sheet of paper. 2. Remind them to show simple solutions (mental or written), focus on the translation rule, and monitor quietly while learners' work. 3. After the activity, discuss answers and solutions step-by-step.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. Provide enrichment tasks such as puzzle cards and GeoGebra challenges for differentiated instruction.	

III. CONTENT	**Lesson 1.3. DILATION (CONCEPT)**
IV. OBJECTIVES	**At the end of the lesson, the learners are will be able to** **1. define dilation as a transformation that changes the size of a figure;** **2. identify the center of dilation in given illustrations;** **3. differentiate enlargement and reduction using scale factor;** **4. describe dilation using appropriate mathematical terms; and** **5. perform dilations on the coordinate plane using a given center and scale factor**.

<table>
<tr><th>V. PROCEDURES</th><th>LEARNERS ACTIVITIES</th><th>ANNOTATION (FACILITATION GUIDE)</th></tr>
<tr><td rowspan="4">A. Activating Prior Knowledge</td><td colspan="2">A.1. Eliciting Prior Knowledge</td></tr>
<tr><td>Activity A.1. LEVELING LEARNER READINESS

Option .. Dilation Brainstorm and Gallery Walk
Instructions:
1. Find a pair and prepare to collaborate.
2. Observe the posted images (e.g., small vs. large banana leaves, miniature and full-size parol, building models).
3. For each image, discuss and record:
a. What properties change between the smaller and larger objects?
b. Which features remain the same?
4. Write your answers on a manila paper/chart.
5. Participate in a gallery walk:
a. Visit other groups' outputs
b. Place a ✓ if you agree or add a short comment.
6. One representative explains the group's observations.

Option 2. Think-Pair-Share: Scaling in Real Life
Instructions:
1. Individually list at least 3 examples of objects that change in size but keep the same shape (e.g., logos, toy cars, resized images).
2. With a partner:
a. Compare your examples
b. Explain why the shape is still the same
3. As a pair, prepare one best example to share with the class.</td><td>Activity A.1. LEVELING LEARNER READINESS
Option 1.
1. Prepare visual examples (preferably localized: parol, banana leaves, Filipino designs).
2. Guide observation using questions:
a. Is the shape still the same?
b. What exactly changed?
3. Encourage use of terms: size, proportion, scaling.
4. During gallery walk, prompt learners to compare

Option 2
1. Give learners thinking time (30–60 seconds).
2. Encourage deeper explanations: Why do you say the shape is preserved?
3. Call selected pairs to share and explain.
4. Affirm correct ideas and refine LEARNER language.

Key Concept to Emphasize: Dilation involves enlargement or reduction, proportional change, and same shape retained.</td></tr>
<tr><td colspan="2">A.2. Establishing the Purpose of the Lesson</td></tr>
<tr><td>Activity A. 2. APPRECIATING LESSON RELEVANCE

Option 1. Predicting Transformations
Instructions:
1. Observe the two images shown by your teacher (a small triangle and a larger triangle).
2. Answer the following questions:
a. What transformation do you think happened?
b. How did the figure change?
3. Write your answer in your notebook using your own words.</td><td>Activity A. 2. APPRECIATING LESSON RELEVANCE

Option 1.
1. Present two similar triangles (same shape, different size).
2. Ask probing questions: what changed? what stayed the same?
3. Guide learners to recognize dilation (size changes, shape preserved).
4. Connect responses to real-life applications (architecture, maps, design).</td></tr>
</table>

	4. Share your answer with a partner, then be ready to share with the class **Option 2. Class Discussion: Dilation in Culture and Nature** **Instructions**: 1. Observe the examples shown by your teacher (e.g., textile patterns, leaves, artworks). 2. Form a group of 3–4 members. 3. Discuss the following questions: a. How did the size change? b. What stayed the same in the design? c. Why do you think this pattern is used? 4. Write your group's answers and be ready to present one idea to the class.	**Option 2.** 1. Use localized examples (e.g., banig, leaf patterns). 2. Encourage learners to relate answers to daily life and culture. 3. Guide learners to the idea of proportional enlargement/reduction. 4. Synthesize answers after group sharing. .
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Option 1. Visual Examples of Dilation** **Instructions**: 1. Observe the given figures showing enlargement and reduction of geometric shapes. ***Dilation*** enlarges or reduces the original figure proportionally from a fixed center, called the center of dilation. The scale factor k determines the change in size. Consider the following figure. 2. Identify the center of dilation, the scale factor, and whether it is an enlargement or reduction. "*What is making the projected image shrink or grow?*" The ***center of dilation*** is a fixed point in the plane about which all points are expanded or contracted. "*How different is one line from the other in the above figure?*" The ***scale factor*** refers to how much the figure grows or shrinks, and it is denoted by:	**Activity B.1. EXPLORING KEY CONCEPTS** **Option 1.** 1. Present clear visual examples of dilation (enlarged and reduced figures). 2. Guide learners using focused questions: a. What point stays fixed? b. What happens to the distance of points from the center? 3. Introduce formal terms gradually: center of dilation, and scale factor. 4. Use GeoGebra or animation to show step-by-step scaling. 5. Emphasize observation before defining formulas. **Option 2.** 1. Show real-life images (preferably Filipino context: maps of barangays, parol models, building plans). 2. Ask guiding questions: a. Which one is bigger or smaller? b. What stayed the same? c. Where is the reference point? 3. Highlight that dilation is commonly used in: a. architecture b. engineering c. map scaling

a. If, the image gets smaller and closer to the center of dilation.
b. If, the image gets larger and farther from the center of dilation.

3. Compare two figures and describe how the size changes.
4. Observe GeoGebra or animated slides showing gradual dilation.

Option 2. Real-World Examples
1. Observe images of maps, building blueprints, and scale models (e.g., landmarks, houses).
2. Answer the following questions:
   a. Which is the original figure?
   b. Which is the dilated image?
   c. Where is the center of dilation?
3. Explain how scaling is shown in each example.

B.2. Discussing the Concept

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Part I. Concept Mapping: Dilation
Instructions:
1. Study the concept of dilation in the table below:

Transformation	Description	Formula
Dilation from the origin	Enlarges or reduces a figure from the origin by a scale factor.	

2. Worked example:
   a. A triangle has vertices: A(2, 1) B(3, 2) C(1, 3)
   b. Dilate the triangle by a scale factor of k = 2 from the origin with center of dilation (1, 2).
   c. The general rule for a dilation about a center (a, b) is $(x, y) \rightarrow (a + k(x - a), b + k(y - b))$.
   d. Find the new coordinates: A', B' C'
3. Follow these steps:
   a. Plot the original triangle.
   b. Plot the center of dilation (1,2).
   c. Draw rays from the center through each vertex.
   d. Locate the new points by doubling the distance from the center.
   e. Draw the new (dilated) triangle.
4. Complete the concept map below:

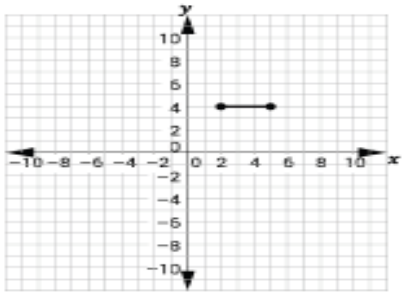
Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Part I.
1. Present the formula and explain briefly before the activity.
2. Emphasize step-by-step plotting and correct use of scale factor.
3. Guide learners who struggle with identifying distances from the center.
4. Use board work for one sample vertex before letting learners proceed independently.
5. The link is used only as enrichment, not as the main task.

Part II.
1. Demonstrate how to use the Dilate tool before the activity.
2. Encourage exploration (trial and error).
3. Facilitate discussion by calling different learners to explain observations.
4. Connect answers to formal mathematical language.

Preview of negative scale factor. A negative scale factor produces a half-turn through the center, with the image on the opposite side of the center. This is an extension; we will treat it briefly in C.4.

No-device alternative. If GeoGebra is unavailable, use graphing paper. Plot the center of dilation and the preimage points, draw

	a. What is dilation? b. What is the role of the scale factor? c. What happens when k>1? d. What happens when 0<k<1? e. How does the center of dilation affect the image? 5. Consider the following graph and show your answers on the board:  a. How do you dilate the line segment on the above graph centered at a point on the same line? b. Use as the center of dilation and complete the following: ● If , then the dilated line segment will have coordinates: _____ and _____. ● If , then the dilated line segment will have coordinates: _____ and _____. ● When dilating a line that passes through the center of dilation, the line is _____. 6. You may visit this link for additional examples: https://thirdspacelearning.com/us/math-resources/topic-guides/geometry/dilations (Focus only on examples of enlargements and reductions.) **Part II. Interactive GeoGebra Exploration** **Instructions**: 1. Open GeoGebra app. 2. Create a triangle or polygon using dilation tool: a. Select the figure b. Choose a center of dilation 3. Perform dilation using different scale factors: a. k = 2 b. k = 1/2 c. k = 3 4. Observe and answer:	rays from the center through each vertex, multiply each distance by k, then plot the image points.

<table>
<tr><td></td><td colspan="2">a. What happens when the scale factor is greater than ?
b. What happens when it is less than ?
c. What happens when you change the center?
5. Share your observations with the class.</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that learners have a copy of the LAS.
Option 1.
1. Group learners in pairs to promote collaboration.
2. Remind learners:
a. If the center is (0,0) → multiply coordinates directly
b. If the center is not the origin → use distance from the center
3. Monitor accuracy in:
a. Plotting points
b. Applying scale factor correctly
4. Provide guided support for struggling learners (remedial): focus only on one vertex at a time.
5. Provide challenge questions for advanced learners (enrichment): try or different centers.

Option 2.
1. Assign realistic contexts to connect math to real life.
2. Encourage creativity but require mathematical accuracy.
3. Guide learners to clearly identify center, scale factor, and type of dilation.
4. During presentation, ask probing questions like: Why did you choose that scale?
5. Differentiation:
a. Struggling learners- provide a guide template.
b. Advanced learners- require coordinate justification.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
1. Choose one base figure: a triangle, a square, or a pentagon.
2. Apply two dilations to your figure: one enlargement with k > 1 and one reduction with 0 < k < 1. Mark the center of dilation clearly for each.
3. Plot the pre-image and the two images on a coordinate grid. For each image, label the center of dilation, the scale factor, and the type (enlargement or reduction).</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS
1. Teacher assigns real-life mini-projects:
a. Poster enlargement
b. Map scaling
c. Model house reduction
2. Pairs identify:
a. center
b. scale factor
c. reason for dilation</td></tr>
</table>

	4. Write a one-paragraph justification of 75 to 100 words. Use the terms scale factor, center of dilation, similar figures, and proportional.	
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Option 1. Group Summary** Instructions: 1. Create a short summary of what you learned about dilation. 2. Write your answers on a manila paper or notebook. 3. Include the following: a. How the shape of a figure is affected b. How the size changes depending on the scale factor c. How to identify the center of dilation 4. Be ready to share your summary with the class. **Option 2.. Class Discussion** **Instruction**: Answer the following questions: 1. What is the mathematical definition of dilation? 2. How do you differentiate enlargement from reduction? 3. State the generalization in your own words.	**Activity C.2. WRAPPING UP THE LESSON** **Option 1.** 1. Encourage learners to explain ideas in their own words. 2. Guide learners to use correct terms like: scale factor, center of dilation, image, and pre-image. 3. Check if learners understand that: shape is preserved and only size and position may change. 4. Call 2–3 pairs to present their work. 5. Provide clarification if misconceptions arise (e.g., confusion between resizing and reshaping). **Option 2.** 1. Facilitate discussion using guided questioning. 2. Encourage multiple learners to respond and build on each other's ideas. 3. Rephrase learner answers into formal mathematical language when needed. 4. Write a final generalization on the board based on LEARNER responses. 5. Ensure all learners understand before closing the lesson.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Emphasize correct use of terms: scale factor, proportional, center of dilation. 2. Watch for misconceptions (e.g., confusing translation with dilation) . 3. Encourage learners to justify answers, not just compute.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. To enhance the learners' skills, facilitate any of the provided activities. 2. Guide learners who need further assistance	

III. CONTENT	Lesson 1.4. DILATION (GRAPHING)	
IV. OBJECTIVES	**At the of the lesson, the learners are will be able to:** **1. determine the scale factor used in dilating a figure;** **2. perform dilation of points and figures on the coordinate plane;** **3. illustrate dilated images correctly using graphing techniques; and** **4. compare corresponding side lengths of original and dilated figures.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS **Option 1. Logo Enlargement** **Instructions**: 1. Work in groups of 4–5. 2. Look at printed simple geometric designs on grid paper (examples: star shape, heart shape, house outline, tree icon, school emblem design (generic, non-branded) 3. Discuss what will happen if each figure is: a. made bigger (enlarged) b. made smaller (reduced) 1. Answer these in your group: a. Which parts of the figure will stay the same? b. Which parts will change? c. Will the shape look different or the same? Why? 2. Write your group answers in 2–3 sentences. **Option 2. Quick Draw Dilation Challenge** 1. Individually, draw a simple figure on grid paper (choose one): a. triangle b. square c. rectangle d. arrow shape 2. Pass your paper to a partner. 3. Your partner will: a. redraw your figure either larger or smaller b. keep the same shape 4. Try to guess: a. Is it an enlargement or reduction? b. What is the approximate scale factor?5. Discuss your reasoning with your partner.	**Activity A.1. LEVELING LEARNER READINESS** **Option 1.** 1. Use non-commercial, neutral figures to avoid brand influence. 2. Emphasize the idea of “same shape, different size”. 3. Guide learners to notice proportional change, not random resizing. 4. Encourage group discussion before writing responses. 5. Facilitate the processing questions. **Option 2.** 1. Check if learners preserve proportionality, not just appearance. 2. Remind learners: dilation is not “freehand resizing.” 3. Encourage estimation of scale factor (no need for exact values at this stage). 4. Walk around to identify misconceptions early. 5. Facilitate the processing questions.

	Processing Questions*:* 1. What happens to a figure when it is enlarged or reduced? 2. How can you tell if one figure is a scaled version of another? 3. What stays the same during dilation?	
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. APPRECIATING LESSON RELEVANCE **Part I. Real-Life Application Discussion** **Instructions***:* 1. The teacher will show images of buildings, city maps, or playground layouts. 2. Answer: How do architects or designers make smaller models of large structures? 3. Discuss how dilation is used in real life for scaling designs accurately. **Part II. GeoGebra Exploration** Instructions: 1. Using GeoGebra, display a triangle and ask learners to predict what will happen if it is dilated by a scale factor. 2. Learners record predictions and later compare with the actual dilation.	**Activity A.2. APPRECIATING LESSON RELEVANCE** 1. Present real-life images such as maps, architectural models, and playground plans. 2. Ask learners to discuss why scaling is important in real-life situations. 3. State the lesson objectives clearly. **Processing Questions**: 1. Why is it important to know the scale factor when creating models or diagrams? 2. How can dilation help in planning and designing real-world structures?
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Logo Enlargement/Reduction** **Instructions** 1. Choose a non-branded geometric figure or a generic Filipino motif (e.g., a simplified parol star, a banig grid pattern, or a Cordillera weave geometric unit). 2. They perform manual dilation on grid paper with two different scale factors. 3. Compare the results and note which aspects remain unchanged (angles, proportionality).	**Activity B.1. EXPLORING KEY CONCEPTS** Demonstrate the step-by-step dilation of a triangle on the coordinate plane using graphing techniques. Have learners replicate the example in pairs and identify corresponding points.
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Part I. Identify Scale Factor**	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** **Part I.**

	Definition (refer to Section IV, Objective a). A dilation with center C and scale factor k is a transformation that maps every point P to a point P′ such that the vector from C to P′ is k times the vector from C to P. In coordinates, if C = (a, b), then P′(x′, y′) = (a + k(x − a), b + k(y − b)). When the center is at the origin, this simplifies to P′(x′, y′) = (kx, ky). When |k| > 1 the figure enlarges; when |k| < 1 the figure reduces; when k = 1 the figure is unchanged. **Instructions***:* 1. Calculate the scale factor of each pair of triangles by comparing their corresponding side lengths. a. Pair 1 (Triangle ABC and *A'B'C'*) b. Pre-image: A(1, 1), B(3, 1), ((2, -4) c. Image: *A'*(2, 2), *B'*(6, 2), *C'*(4, 8) d. Pair 2 (Triangle DEF and *D'E'F'*) e. Pre-image: D(2, 2), E(6, 2), F(4, 6) f. Image: *D'*(1, 1), *E'*(3, 1), *F'*(2, 3) 2. Guide: a. Compare matching sides (AB with *A'B'*, BC with *B'C'*, etc.) b. Write the ratio: Image side ÷ Original side **Part II. Corresponding Side Comparison** Solve the problem on the board. Show how you matched corresponding sides and how you computed the scale factor. Problem 1. Find the scale factor and state if it is an enlargement or reduction. • Triangle PQR has sides: PQ = 4, QR = 6, RP = 8 • Triangle *P'Q'R'* has sides: *P'Q'* = 8, *Q'R'* = 12, *R'P'* = 1 Problem 2. Find the scale factor and type of dilation. • Triangle XYZ has sides: XY = 10, YZ = 5, ZX = 10 • Triangle *X'Y'Z'* has sides: *X'Y'* = 5, Y'Z' = 2.5, *Z'X'* = 5 **Part III. GeoGebra Interactive Example** *Instructions:* 1. Open GeoGebra and plot the given figure on the coordinate plane. 2. Use the center of dilation and scale factor provided. 3. Drag or transform the figure to create its image. 4. Observe what changes and what stays the same after dilation.	1. Guide learners to ensure correct identification of corresponding vertices before solving. 2. Remind learners that scale factor is always image ÷ original, not the reverse. 3. Highlight consistency across all side pairs to confirm correctness. 4. Address misconception: “Different answers mean incorrect pairing of vertices.” **Part II.** 1. Encourage learners to show step-by-step reasoning, not just final answers. 2. Emphasize checking at least two pairs of sides to confirm accuracy. 3. Reinforce proportional reasoning as foundation of dilation. 4. Facilitate board sharing to promote mathematical communication. 5. Watch for errors in mismatched correspondence (most common issue). **Part III.** 1. Ensure all learners are logged into GeoGebra and can access the coordinate plane before starting. 2. Briefly demonstrate how to: • Plot points using coordinates • Set or identify the center of dilation • Use the scale factor tool/slider (if available) 4. Remind learners that accuracy in plotting is essential for correct results. *During Task Implementation:* Task 1 (Enlargement,): a. Guide learners to observe that the image moves farther from the center of dilation. b. Reinforce that all corresponding sides should double in length. c. Ask: “What do you notice about the distance from the origin after dilation?” Task 2 (Reduction,):

<table>
<tr><td></td><td>Task 1: Triangle Dilation (Enlargement)
Figure: Triangle ABC
Coordinates: A(1, 1), B(3, 1), C(2, 4)
Center of Dilation: O(0, 0)
Scale Factor: 2

Task 2: Triangle Dilation (Reduction)
Figure: Triangle DEF
Coordinates: D(4, 4), E(8, 4), F(6, 10)
Center of Dilation: O(0, 0)
Scale Factor: 1/2

Task 3: Quadrilateral Dilation (Enlargement)
Figure: Quadrilateral PQRS
Coordinates: P(1, 2), Q(3, 2), R(3, 2), S(1, 5)
Center of Dilation: O(1, 1)
Scale Factor: 3</td><td>a. Emphasize that the image should move closer to the center of dilation.
b. Watch for learners mistakenly enlarging instead of reducing.
c. Prompt: “How does halving the scale factor affect the size of the figure?”

Task 3 (Enlargement with non-origin center):
a. Highlight the importance of the center of dilation not always being (0,0).
b. Learners may incorrectly apply scaling from the origin; correct this immediately.
c. Ask: “What changes when the center is not at the origin?”

Processing Questions:
1. What happens to the size of the figure?
2. What happens to the position of the figure?
3. What stays the same after dilation?
4. How does the center of dilation affect the image?</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.
Option 1.
1. Emphasize correct use of center of dilation (most common error point).
2. Remind learners of the 3-step process:
a. Subtract center
b. Multiply by scale factor
c. Add center back
3. Circulate and check if learners are correctly scaling each vertex consistently.
4. Encourage peer checking before final submission.

Option 2.
1. Emphasize that reverse dilation requires division, not multiplication.
2. Reinforce checking by verifying all corresponding points.
3. Ask guiding questions:
a. What operation undoes dilation?
b. How do you confirm your answer is correct?
4. Watch for confusion between enlargement and reduction.</td></tr>
<tr><td></td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS</td></tr>
</table>

C. Demonstrating Knowledge and Skills	**Option 1 Architecture/Design Connection** **Instructions**: 1. Look at the pictures of real-life structures shown by your teacher. 2. Study each image carefully. 3. Answer the questions below for each structure. a. If you were to create a model of this structure, would you make it larger or smaller? Why? b. What scale factor do you think is appropriate (e.g., 1/2, 2, 3)? c. What stays the same even if the model is resized? d. What changes when the structure is scaled? **Option 2. Map Scaling Problem** **Instructions**: 1. Study the map given to you. 2. Your task is to enlarge the map using the given scale factor. 3. Draw the new enlarged map on graph paper. 4. Compare your answer with your classmates. 5. Given: Original Map Grid • Points S1(1, 1, S2(1, 2) – School • Points H1(3, 1), H2(3, 3) – Hospital • Points M1(3, 6), M(3, 7) – Market • Points P1(1, 3), P2(1, 5) - Park • Scale Factor: 5 6. Do the tasks: a. Multiply each coordinate by b. Draw the enlarged map on graph paper c. Label all points clearly	*Note:* 1. You may use the provided figures OR replace/add other appropriate real-life images such as local landmarks, school buildings, community structures, or classroom objects. 2. Ensure that selected images are familiar and culturally relevant to learners. 3. Emphasize that the goal is to connect dilation to real-world scaling and proportionality. **Option 1.** 1. This activity develops learners' understanding of dilation in real-life contexts (architecture and design). 2. Guide learners to recognize that: • Shape remains the same • Size changes depending on scale factor 3. Expected responses are qualitative (no exact computation required). 4. Encourage discussion rather than strict numerical answers. **Option 2.** 1. Remind learners that enlargement affects distance proportionally, not shape. 2. Check if learners correctly apply multiplication to both x and y coordinates. 3. Emphasize accuracy in plotting points on graph paper. 4. Ask learners to explain how proportionality is reflected in their drawing.
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Option 1. Dilation Rules Summary** 1. Answer the questions below in your own words. a. Write your answers in complete sentences.	**Activity C.2. WRAPPING UP THE LESSON** **Option 1.** 1. Emphasize that learners should use their own understanding, not memorization.

b. What stays the same when a figure is dilated?
c. What changes when a figure is dilated?

2. After answering, contribute one idea to the class summary chart on the board.
3. Class Summary Chart (to be completed together):

What Stays the Same?	What Changes?

Option 2. Concept Mapping

1. Work in groups of 4–5.
2. Create a concept map using the words below:
   a. Scale Factor
   b. Dilation
   c. Side Lengths
   d. Center of Dilation
   e. Enlargement
   f. Reduction
   g. Real-life Applications
   h. Art and Design
3. Show how these ideas are connected using arrows, lines, or linking words.
4. Present your concept map to the class.

2. Guide learners to distinguish between invariant properties (angles/shape) and variable properties (size/lengths).
3. Use LEARNER responses to correct misconceptions (e.g., "shape changes" is incorrect).
4. Facilitate class chart building as a collaborative synthesis activity.

Generalization. In a dilation, corresponding angles stay equal, side lengths are multiplied by k, and distances from the center are multiplied by k. For $k > 0$, orientation is preserved. If k is not 1, the image is similar but not congruent to the preimage.

Option 2.

1. Encourage creativity in layout (no single correct format).
2. Check for correct conceptual links, not artistic quality.
3. Support learners in using linking words like: affects, leads to, results in, preserves.
4. Address misconceptions such as dilation changes shape or angles change.
5. Allow group discussion before final output to strengthen conceptual understanding.

C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)

Activity C.3. ASSESSING LEARNING OUTCOMES

Before facilitating this activity, ensure that all learners have a copy of the LAS.

1. Introduce the activity by explaining that learners will apply their knowledge of dilation to solve problems involving scale factors, coordinates, and side lengths.
2. Distribute the assessment worksheet and ensure each learner has graph paper and writing materials.
3. Instruct learners to read each item carefully and show complete solutions, including labeled diagrams on graph paper when necessary.
4. Guide learners to work individually while allowing clarification questions when needed, but avoid giving direct answers.
5. Remind learners to identify corresponding points, sides, and scale factors clearly before solving each problem.
6. Monitor learners as they perform dilation on the coordinate plane, ensuring proper plotting and accuracy.
7. Encourage learners to explain their reasoning, especially when determining scale factors and comparing side lengths.
8. After completion, facilitate a short sharing session where selected learners explain their answers and strategies for solving the tasks.

	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Begin by introducing the activity and explaining that learners will use GeoGebra to explore dynamic dilations of polygons and geometric figures. 2. Guide learners in accessing and navigating GeoGebra, ensuring they can construct basic shapes and apply the dilation tool correctly. 3. Instruct learners to create and observe how shapes change under different scale factors, emphasizing how size changes while shape and proportions remain the same. 4. Provide an optional challenge where learners create and dilate simple Filipino-inspired geometric motifs (e.g., parol star, banig grid pattern, or Cordillera-inspired weave designs), and compute the corresponding scale factors. Ensure learners avoid reproducing copyrighted or registered logos. 5. Facilitate a discussion on real-life applications of scaling and proportionality, such as in architecture, clothing design, maps, and sports field layouts. Encourage learners to share examples from their own experiences. 6. For advanced learners, introduce the concept of negative scale factors and explain that it results in a transformation involving a 180-degree rotation or point reflection about the center. 7. Guide advanced learners through the example of dilating triangle XYZ with k = −1 about the origin, and ask them to describe and interpret the resulting image. 8. Conclude by asking learners to reflect on how technology helped them better understand dilation and its applications in real-world contexts.

III. CONTENT	**Lesson 1.5. ROTATION**	
IV. OBJECTIVES	**At the end of the lesson, the learners are will be able to** **1. define rotation as a transformation that turns a figure about a fixed point;** **2. identify the center of rotation in given situations;** **3. determine the angle and direction of rotation (clockwise or counterclockwise); and** **4. illustrate rotation of geometric figures on the coordinate plane.**	
V. PROCEDURES	**LEANERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS **Part I. Rotation in Real Life (Observation and Discussion)** 1. Form a group of 4–5 members. 2. Observe the objects shown by your teacher or look around your classroom/environment. 3. Study the following examples: a. Clock b. Electric fan	**Activity A.1. LEVELING LEARNER READINESS** **Part I.** 1. Use familiar, non-branded real-life objects to make the concept relatable. 2. Guide learners to identify the center of rotation clearly. 3. Emphasize that rotation involves: a. turning around a fixed point b. no change in size or shape

c. Ferris wheel (picture)
d. Pinwheel

4. Discuss the questions below with your group:
   a. Which objects move in a circular or turning motion?
   b. What point do these objects rotate around?
   c. Does the object change its shape while rotating?
5. Write at least 2 examples and your answers on paper.
6. Share your answers with the class.

Part II. Quick Hands-On Rotation

1. Stay in your group.
2. Get a cut-out shape (triangle or square) and place it on your table.
3. Mark a point on the table (this will be the center of rotation).
4. Rotate the shape around the marked point:
   a. First, turn it clockwise
   b. Then, turn it counterclockwise
5. Observe and discuss:
   a. What direction did the shape move?
   b. Did the shape change in size or form?
   c. Where is the center of rotation?
6. Record your observations.

Processing Questions:

1. Which objects or shapes in your discussion rotate around a fixed point?
2. How do clockwise and counterclockwise turns differ?

4. Allow learners to share different examples to encourage participation.
5. This activity serves as a diagnostic tool to check prior knowledge.

Part II.

1. Demonstrate rotation first before letting learners try.
2. Ensure learners rotate around the correct fixed point (common error: sliding instead of rotating).
3. Emphasize difference between:
   a. Rotation (turning)
   b. Translation (sliding)
4. Ask guiding questions:
   a. Did the shape stretch or change?
   b. What stayed the same?
5. Use this activity to introduce key terms: center of rotation, clockwise, counterclockwise

A.2. Establishing the Purpose of the Lesson

Activity A. 2. APPRECIATING LESSON RELEVANCE

Option 1. Why Study Rotation?

1. Look at the images shown by your teacher (e.g., windmill, flower petals, parol, circular patterns).
2. Work with a partner.
3. Complete the table below:

Object/ Design	Where do you see rotation?	What is the center of rotation?	Why is rotation important here?

Activity A. 2. APPRECIATING LESSON RELEVANCE

Option 1.

1. Ensure learners identify both rotation AND the center, not just the object.
2. Encourage use of real-life reasoning (design, balance, motion).
3. Accept varied answers but check for correctness of concept.
4. You may add more local or culturally relevant images (e

Option 2.

1. Pause before rotating to allow LEARNER predictions (builds engagement).
2. Emphasize:

	4. Write at least 2 examples. 5. Prepare a 1–2 sentence explanation answering how rotation is used in real-life designs or objects. 6. Share your answers with the class. **Option 2. GeoGebra Demonstration** Watch the teacher rotate a geometric design twice. Note that the design returns to a familiar shape after a few rotations. Why might this idea be useful in art and architecture? You will study how to perform and predict rotations in the lessons that follow.	a. Rotation = turning around a fixed point b. Angle determines how far the figure moves 3. Ask guiding questions: a. Why did the point move there? b. What stayed the same after rotation? 4. Clarify misconceptions (e.g., learners thinking size changes). 5. You may let learners manipulate GeoGebra themselves if resources allow.
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS Teacher Demonstration: Rotation Rules about the Origin. For rotations about the origin, use these rules: 90° CCW: (x, y) → (−y, x) 90° CW: (x, y) → (y, −x) 180°: (x, y) → (−x, −y) 270° CCW (equivalent to 90° CW): (x, y) → (y, −x) Worked example. If A(3, 4), then $R_{+90}°(A) = (-4, 3)$; $R_{-90}°(A) = (4, -3)$; $R_{180}°(A) = (-3, -4)$; $R_{270}°(A) = (4, -3)$. **Option 1. Rotation on Paper** 1. Work in groups of 3 to 4. 2. Plot the given figure on graph paper. Figure: Triangle with points and center of rotation at 3. Perform the following rotations and draw each image on the same graph: a. Rotate 90^0clockwise. b. Rotate 180^0. c. Rotate 90^0 counterclockwise. 4. Label the new points as *A′, B′, C′* for each rotation. 5. Compare your drawings with your groupmates. **Option 2. GeoGebra Exploration** 1. Open GeoGebra. 2. Plot the triangle below: 3. Center of rotation: 4. Perform the following rotations: a. Rotate counterclockwise	**Activity B.1. EXPLORING KEY CONCEPTS** **Option 1.** 1. Demonstrate one rotation before letting learner's work. 2. Emphasize that: a. Rotation keeps size and shape the same b. Only position changes 3. Common errors: a. Learners reflect instead of rotate b. Incorrect sign changes 4. Guide learners to recognize rotation rules/patterns. 5. Encourage checking answers using ordered pairs. **Option 2.** 1. Let learners predict before clicking rotate. 2. Emphasize connection between: ● angle of rotation ● position of image 3. Ask guiding questions: ● Where did the point move? ● What pattern do you notice? 4. Ensure learners understand that center of rotation affects the result. 5. If possible, allow learners to manipulate GeoGebra themselves.

b. Rotate

5. Observe and complete the table:

Rotation	New Coordinates of Points	What changed?	What stayed the same?
90° CCW			
180°			

Processing Questions:

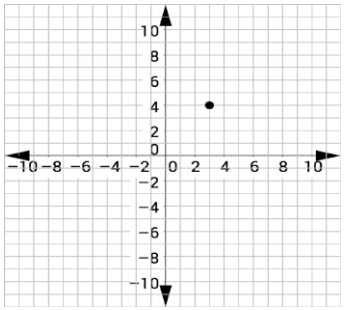
1. What is the center of rotation in each example?
2. How does the angle of rotation affect the new position of the figure?

B.2. Discussing the Concept

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Definition. A rotation is a rigid motion that turns every point of a figure through a given angle around a fixed point called the center of rotation. The image is congruent to the preimage. The distance of every point from the center stays the same. Size, shape, and angle measures are preserved. For rotations about the origin, the four rules above (90° CCW, 90° CW, 180°, 270° CCW) apply.

A **rotation** changes the position of a figure by moving it around a fixed point through a specified angle. A clockwise rotation turns the figure in the same direction as the hands of a clock; a counterclockwise rotation turns it in the opposite direction. By mathematical convention, a positive angle measure denotes counterclockwise rotation, and a negative angle measure denotes clockwise rotation. Consider the graph below.



Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

1. Introduce rotation as a rigid transformation that turns a figure around a fixed point called the center of rotation.
2. Explain that during rotation, the figure maintains its size, shape, side lengths, and angle measures because rotation is a rigid motion.
3. Discuss the meaning of clockwise and counterclockwise rotation, emphasizing that:
4. positive angle measures indicate counterclockwise rotation
5. negative angle measures indicate clockwise rotation
6. Review the standard rotation rules about the origin and relate them to the movement of points on the coordinate plane.

$$90^\circ\ CC: (x, y) \rightarrow (-y, x)$$
$$90^\circ\ CW: (x, y) \rightarrow (y, -x)$$
$$180^\circ: (x, y) \rightarrow (-x, -y)$$

7. Present the graph and guide learners in analyzing how the coordinates change after rotation about the origin.
8. Ask learners to complete the statements:
   a. The function $R_t(x, y)$ rotates the point (x, y) t° _____ around the origin.
   b. The function $R_{-t}(x, y)$ rotates the point (x, y) t° _____ around the origin.
9. Facilitate a discussion on the following key concepts:
   - **Center of rotation** – the fixed point around which the figure turns
   - **Angle of rotation** – the degree measure of the turn
   - **Direction of rotation** – clockwise or counterclockwise

Use the graph to help you determine the coordinates for (x', y') after the following rotations about the origin.

Degrees	Counter Clockwise	Clockwise
90^0 Rotation	$R_{90^0}(3, 4)$ =_____	$R_{-90^0}(3, 4)$ =_____
180^0 Rotation	$R_{18^0}(3, 4)$ =_____	$R_{-18^0}(3, 4)$ =_____
270^0 Rotation	$R_{27^0}(3, 4)$ =_____	$R_{-27^0}(3, 4)$ =_____

The function rotates the point _____ around the origin. The function rotates the point _____ around the origin.

2. Discuss key concepts and answer the items below:
   a. Concepts:
      - Center of rotation – the fixed point around which the figure turns
      - Angle of rotation – degree measure of turn
      - Direction of rotation - clockwise or counterclockwise
   b. Tasks:
      - Segment RT has endpoint R(0, 3) and T(4, 1). Rotate segment RT 90^0 about the origin. Write an algebraic description of the transformation of segment RT. What are the endpoints of the new line segment?
      - Consider the graph above, rotate (3, 4) counterclockwise 900 about (5, 7)

10. Encourage learners to explain how the angle and direction of rotation affect the new position of a figure.
11. Ask learners to compare the preimage and image and identify which properties remain unchanged after rotation.
12. Conclude the activity by emphasizing that rotation changes the orientation and position of a figure while preserving its size and shape.

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

Activity B.3.
PRACTICING LEARNED SKILLS: ROTATING CUT OUTS

Before facilitating this activity, ensure that all learners have a copy of the LAS.
Using cut-outs and GeoGebra let learners compare the predicted and actual rotated positions. Guide learners in accomplishing the task.

C. Demonstrating Knowledge and Skills

C.1. Finding Practical Application

Activity C.1
MAKING REAL-WORLD CONNECTIONS: ROTATIONS IN FILIPINO ART

Instructions*:*
1. Observe the Filipino-inspired designs shown by your teacher (e.g., parol, circular textile patterns, mandala-style designs).
2. Choose one design and study it carefully.
3. Complete the table below:

Activity C.1
MAKING REAL-WORLD CONNECTIONS: ROTATIONS IN FILIPINO ART

1. Provide clear, non-branded Filipino designs (printed or projected).
2. Guide learners to identify repeating patterns around a central point.
3. Accept approximate angles but ensure reasoning is correct.

Design Observed	Center of Rotation	Angle of Rotation (estimate)	Direction (CW/CCW)	How rotation creates balance

4. Answer the question in 1–2 sentences:
   How does rotation help make the design balanced and visually appealing?
5. Share your answer with the class.

4. Emphasize connection between rotation and symmetry in art.

Note: Teacher may include other culturally relevant designs (e.g., banig patterns, okir designs).

Processing Questions:
1. How does rotation help create patterns in art?
2. Why is identifying the center of rotation important in real-life objects?

C.2. Making Generalization

Activity C.2:
WRAPPING UP THE LESSON: ROTATION RULES TABLE

Instructions:
1. Create a table summarizing:
   a. Type of rotation
   b. Center of rotation
   c. Angle
   d. Direction
   e. Example in real life or art
2. Generalize about rotations around the origin to complete the following table.

Degree	Counterclockwise	Clockwise
90° Rotation	(x, y) →	(x, y) →
180° Rotation	(x, y) →	(x, y) →
270° Rotation	(x, y) →	(x, y) →

Activity C.2:
WRAPPING UP THE LESSON: ROTATION RULES TABLE

1. Instruct learners to create a table containing the following information:
   a. type of rotation
   b. center of rotation
   c. angle of rotation
   d. direction of rotation
   e. real-life or artistic example of rotation
2. Guide learners in recalling the standard rotation rules about the origin and how coordinates change after rotation.
3. Ask learners to complete the Rotation Rules Table by writing the appropriate coordinate rule and applying it to the example point A(2,1).

$$90^\circ\ CCW: (x, y) \rightarrow (-y, x)$$
$$180^\circ: (x, y) \rightarrow (-x, -y)$$
$$90^\circ\ CW: (x, y) \rightarrow (y, -x)$$
$$360^\circ: (x, y) \rightarrow (x, y)$$

4. Encourage learners to analyze patterns in the coordinate changes for different angles and directions of rotation.
5. Facilitate discussion by asking learners:
   a. What similarities and differences do you observe among the rotation rules?

		b. Why does a 360° rotation return the figure to its original position? 6. Allow learners to share real-life examples where rotation can be observed in objects, art, and designs. 7. Conclude the lesson by emphasizing that rotation changes the position and orientation of a figure while preserving its size and shape
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Distribute the mini-quiz and provide learners with graph paper and writing materials. Remind them to show complete solutions and clearly label the origin $O(0,0)$ in their drawings. 2. Instruct learners to answer Part I individually by applying the correct rotation rules to determine the image coordinates and graph the preimage and image on the coordinate plane. 3. Monitor learners as they solve the problems, guiding them to identify the angle and direction of rotation before performing each transformation. 4. Organize learners into groups of 3–4 for Part II and instruct them to solve the rotation challenge collaboratively and explain their reasoning. 5. Facilitate a short discussion where selected learners or groups present their answers and explain how they identified the center, angle, and direction of rotation.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. ENHANCING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Emphasize invariance: same shape, same size 2. Ask what happens after a full turn? 3. Allow learners to explore beyond given Instructions: (e.g., try different angles). 4. Ensure learners correctly apply rotation rules.	

III. CONTENT	**Lesson 1.6. REFLECTION**
IV. OBJECTIVES	**At the end of the lesson, the learners are will be able to:** **1. define reflection as a transformation producing a mirror image;** **2. identify the line of symmetry in given figures;** **3. perform reflection across the x-axis, y-axis, and other lines; and** **4. Illustrate reflected images accurately on the coordinate plane.**

<table>
<tr><th>V. PROCEDURES</th><th>LEARNERS ACTIVITIES</th><th>ANNOTATION (FACILITATION GUIDE)</th></tr>
<tr><td rowspan="4">A. Activating Prior Knowledge</td><td colspan="2">A.1. Eliciting Prior Knowledge</td></tr>
<tr><td>Activity A.1. LEVELING LEARNER READINESS

Option I. Mirror Folding Challenge
Instructions:
1. Work in groups of 3–4 members.
2. You will be given printed shapes on graph paper.
3. For each shape, do the following:
a. Look at the figure carefully.
b. Predict a possible line of symmetry (draw a dotted line).
c. Fold the paper along your predicted line.
d. Check if both sides match perfectly.
e. If it does not match, try another line.
4. Answer in your group:
a. Was your prediction correct?
b. What made the shape symmetrical or not symmetrical?
5. Choose one shape and present your answer to the class.

Option 2. Digital Exploration Using GeoGebra
1. Open GeoGebra on a computer or tablet.
2. Select or load a simple shape (triangle, rectangle, or star).
3. Identify a possible line of symmetry on the shape.
4. Use the reflection tool to reflect the figure across your chosen line.
5. Observe the result carefully.
6. Answer:
a. Does the reflected image match the original shape?
b. Was your predicted line of symmetry correct?
7. Discuss your findings with your group.

Processing Questions:
1. How did folding or using a mirror help you?
2. What do you notice about symmetrical shapes?
3. Why is symmetry important in designs?</td><td>Activity A.1. LEVELING LEARNER READINESS

Option 1.
1. Provide simple geometric figures (triangle, square, heart, star).
2. Emphasize that symmetry means exact matching halves.
3. Guide learners away from guessing and toward visual verification (fold test).
4. Ask: “Do both sides match exactly?”
5. Clarify misconception: approximate similarity is NOT symmetry.

Option 2.
1. Ensure learners correctly use the reflection tool in GeoGebra.
2. Encourage prediction BEFORE testing.
3. Reinforce concept: symmetry = mirror image across a line
4. Check common misconception: learners think any dividing line is a line of symmetry.
5. Allow learners to retry multiple times for discovery learning.

If GeoGebra is unavailable, substitute graph paper with a ruler and the coordinate rules.</td></tr>
<tr><td colspan="2">A.2. Establishing the Purpose of the Lesson</td></tr>
<tr><td>Activity A.2
APPRECIATING LESSON RELEVANCE:
REAL-LIFE REFLECTION HUNT</td><td>Activity A.2
APPRECIATING LESSON RELEVANCE:
REAL-LIFE REFLECTION HUNT</td></tr>
</table>

<table>
<tr><td></td><td>Instructions:
1. Work with a partner.
2. Look around your classroom or school environment.
3. Identify at least 3 objects that show reflection symmetry.
4. Examples you may observe:
a. windows
b. doors
c. floor tiles
d. whiteboard/posters
e. lockers or cabinets
5. For each object, complete the table below:<table><tr><th>Object Found</th><th>Line of Symmetry (if visible)</th><th>Why is it symmetrical?</th></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>6. Discuss with your partner:
a. How does symmetry affect the appearance of the object?
b. Why do designers use symmetry?
7. Share one example with the class.

Processing Questions:
1. Why do we need to know the line of symmetry before performing reflection?
2. How does reflection relate to patterns in nature and design?</td><td>1. Encourage learners to use real classroom objects (not imaginary ones).
2. Emphasize that symmetry is based on visual equality of halves.
3. Accept multiple correct answers depending on observation.
4. Guide learners to the discussion.
a. natural symmetry (tiles, windows)
b. designed symmetry (posters, boards)

Connection to the Performance Task. In the unit Math-Art Exhibit titled The Hidden Geometry of Design, the Transformation Specialist on each curatorial team must produce a Transformation Poster showing one base figure transformed by each of the five named transformations: translation, reflection, rotation, dilation, and glide reflection. The reflection step uses exactly the techniques you are learning today (line of symmetry, equidistance, and reflection across the x-axis, y-axis, y = x, and y = −x).</td></tr>
<tr><td rowspan="2">B. Instituting New Knowledge</td><td colspan="2">B.1. Presenting Examples</td></tr>
<tr><td>Activity B.1. EXPLORING KEY CONCEPTS

Instructions:
Familiarize the rules below and apply to the indicated tasks.

Reflection rules.
1. Across the x-axis: $(x, y) \rightarrow (x, -y)$.
2. Across the y-axis: $(x, y) \rightarrow (-x, y)$.
3. Across $y = x$: $(x, y) \rightarrow (y, x)$.
4. Across $y = -x$: $(x, y) \rightarrow (-y, -x)$.

Part I. Reflection Across Axes
1. Plot the points A(3, 2), B(−2, 4), and C(1, 5) on the coordinate plane.</td><td>Activity B.1. EXPLORING KEY CONCEPTS

Part I.
1. Clearly demonstrate at least one reflection per axis before learner work.
2. Emphasize transformation rules (do NOT let learners guess randomly).
3. Highlight common misconception: learners often swap signs incorrectly
4. Encourage learners to verify answers using GeoGebra if available.
5. Ask guiding questions:
a. What changed in the coordinates?
b. What pattern do you notice for each axis?</td></tr>
</table>

2. Reflect A across the x-axis, B across the y-axis, and C across the line y = x. Use the rules from B.2 to verify your answers.
3. Complete the table below by recording the coordinates before and after reflection.
4. Answer the following:
   a. What changes after reflection?
   b. What stays the same?

Point/ Shape	Reflection	Original Coordinates	Reflected Coordinates
A	x-axis		
B	y-axis		
C	y = x		

Part II. Grid Work Exercise

1. Work in groups of 3–4.
2. You will be given graph paper with shapes already drawn.
3. For each shape:
   a. Draw a possible line of symmetry or reflection line
   b. Predict the reflected image
   c. Draw the reflected figure on the graph
   d. Verify your answer using folding (if paper allows) or re-checking on the grid
4. Compare your work with your groupmates.
5. Present one example to the class.

Processing Questions

1. What happens to the x-coordinate when reflecting across y-axis?
2. What stays the same after reflection?

Part II.

1. Provide simple geometric shapes (triangle, square, letter shapes, or irregular polygons).
2. Ensure learners:
   a. correctly identify reflection line
   b. plot accurate mirrored points
3. Emphasize:
   a. reflection = mirror image
   b. distance from line of reflection must be equal on both sides
4. Common error: learners shifting shape instead of reflecting it
5. Encourage peer checking before final submission

B.2. Discussing the Concept

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Instructions:

1. Study the definition of reflection as a transformation producing a mirror image.
2. Explain lines of symmetry and their role in determining reflected images.
3. Discuss equal distances of points from the mirror line.

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

1. Emphasize that reflection produces a mirror image, not a resized or rotated figure.
2. Clarify that: distance from the line of reflection is always equal on both sides.
3. Common misconception: learners think only signs change randomly.
4. Reinforce systematic rules for each axis/line.

A ***reflection*** is a rigid transformation that produces a mirror image of a figure across a line called the *line of reflection*. A reflection preserves size, shape, side lengths, and angle measures, so the image is congruent to the preimage. However, the *orientation is reversed*. Each point and its image are the same perpendicular distance from the line of reflection.

The function $r_L(x, y)$ denotes the reflection of the point (x, y) across the line L. For example, $r_{x\text{-}axis}(3, 2)$ denotes the reflection of (3, 2) across the x-axis. Computing the four reflections of (3, 2): $r_{x\text{-}axis}(3, 2) = (3, -2)$; $r_{y\text{-}axis}(3, 2) = (-3, 2)$; $r_{y=x}(3, 2) = (2, 3)$; $r_{y=-x}(3, 2) = (-2, -3)$.

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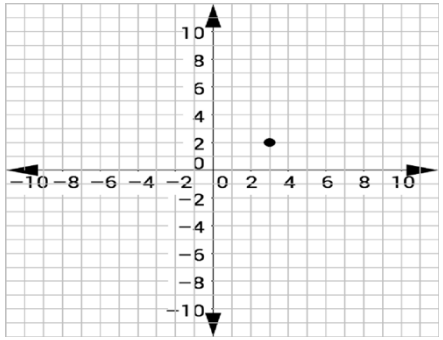


Reflection over	Notation	New coordinates
x-axis	$r_{x-axis}(3, 2)$	
y-axis	$r_{y-axis}(3, 2)$	
$y = x$	$r_{y=x}(3, 2)$	
$y = -x$	$r_{y=-x}(3, 2)$	

Reflection Rules:
Use the guide below to understand how reflection works:

1. Reflection over x - axis: $(x, y) \rightarrow (x, -y)$
2. Reflection over y - axis: $(x, y) \rightarrow (-x, y)$
3. Reflection over $y = x$: $(x, y) \rightarrow (y, x)$
4. Reflection over $y = -x$ $(x, y) \rightarrow (-y, -x)$

5. Encourage learners to check patterns across all four reflections.

Note on terminology. A *line of reflection* is the mirror line used to reflect a figure to a new location. A *line of symmetry* is a special line of reflection that maps a figure onto itself. When reflecting a figure to a new location, use the term *line of reflection*. When checking whether one figure folds onto itself, use the term *line of symmetry*

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

<table>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.
Part I.
1. Ensure learners correctly identify: x-axis → flip up/down and y-axis → flip left/right
2. Emphasize that reflection keeps shape and size unchanged.
3. Common errors: shifting instead of reflecting, incorrect sign changes.
4. Provide simple geometric or abstract designs (no brand logos)
5. Allow peer checking before final submission

Part II.
1. Ensure learners use correct reflection rules consistently.
2. Require graphing for Problem 3 (objective d).
3. Check correct coordinates, correct orientation, and correct labeling.
4. Reinforce that reflection does NOT change shape or size</td></tr>
<tr><td rowspan="4">C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: DESIGN CHALLENGE

Instructions
1. Choose one Filipino-inspired geometric pattern shown by your teacher (e.g., parol design, textile pattern, or circular motif).
2. On graph paper, draw the original design.
3. Reflect the design across a chosen line of symmetry (horizontal, vertical, or diagonal).
4. Draw the reflected image clearly and label both figures:
a. Original figure (Pre-image)
b. Reflected figure (Image)
c. Line of symmetry
5. Answer the following in 2–3 sentences:
a. What type of reflection did you use?
b. How does reflection make the design more visually appealing?</td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: DESIGN CHALLENGE

1. Provide simple Filipino-inspired geometric designs (not brand-related).
2. Emphasize: reflection creates mirror-image symmetry
3. Check correct line of symmetry, and accurate mirrored points.
4. Encourage creativity while maintaining mathematical accuracy
5. Ask guiding question: Are both sides exactly equal in shape and size?

Connection to the Performance Task. This design can be used as one part of the Transformation Specialist poster for the Math-Art Exhibit. In your written explanation, state the line of reflection and explain why the reflected design is congruent to the original.</td></tr>
<tr><td colspan="2">C.2. Making Generalization</td></tr>
<tr><td>Activity C.2. WRAPPING UP THE LESSON

Instructions:
1. Think about everything you learned about reflection today.
2. Work individually first and complete the sentence starter below:
Reflection is _____.</td><td>Activity C.2. WRAPPING UP THE LESSON
1. Guide learners to use correct mathematical terms:
a. mirror image
b. line of symmetry
c. equidistant
2. Clarify misconception: reflection is NOT resizing or rotating.</td></tr>
</table>

	3. In pairs, compare your answers. 4. As a class, help complete the summary chart on the board: a. **Class Summary Guide:** Complete the key ideas below. a. Reflection produces _____. b. Lines of symmetry help us _____. c. Reflected points are _____ from the mirror line. 5. Be ready to share one important idea you learned today.	3. Emphasize key idea: distance from mirror line is always equal on both sides 4. Use LEARNER responses to check understanding before moving to assessment 5. Encourage all learners to contribute at least one idea **Operational characterization.** In any reflection, corresponding points lie on opposite sides of the line of reflection, the segment joining each point to its image is perpendicular to the line of reflection, and the two points are the same distance from the line.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOME Before facilitating this activity, ensure that all learners have a copy of the LAS. Let learner perform Activity 2 on the LAS. Provide assistance and guidance while learners performing the task.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. ENHANCING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a **copy** of the LAS. 1. Prepare simple geometric figures (avoid logos or branded designs) 2. Ensure fairness by giving equal difficulty sets Reinforce: a. reflection = mirror image b. equal distance from line of symmetry 3. Monitor teamwork and participation 4. Use this as a formative assessment + engagement activity.	

III. CONTENT	**Lesson 1.7. TILING AND TESSELLATION**	
IV. OBJECTIVES	**At the end of the lesson, learners are will be able to:** **1. define tessellation;** **2. identify polygons that can tessellate the plane (e.g., triangles, squares, hexagons);** **3. generate tessellations using geometric transformation;** **4. describe the vertex configuration of a tessellation and justify the tessellation using an interior-angle computation; and** **5. explain how translation, rotation, or reflection generates a tessellation with no gaps and no overlaps.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNERS READINESS	**Activity A.1. LEVELING LEARNERS READINESS** 1. You will present pictures of real-life patterns such as floor tiles, brick walls, honeycombs, mosaics, and textile designs,

	The teacher will show pictures of real-life patterns such as floor tiles, brick walls, honeycombs, and mosaic and textile designs, then ask learners to observe the patterns and respond to the questions. **Processing Questions**: 1. What shapes can you identify in the pictures? 2. Do the shapes repeat in a pattern regularly? 3. Do you notice any gaps or overlaps? 4. Based on your observations, what do you think this kind of pattern is called? **Part II. Exploring Polygons That Fit Together** 1. Divide the class into small groups. 2. Each team is given cut-out polygons (triangles, squares, pentagons, hexagons). 3. Each team is instructed to arrange the shapes on a sheet of paper and attempt to cover the surface completely without leaving gaps or overlaps. 4. After the activity, each team shares their observations.	then ask learners to carefully observe the arrangement of shapes and identify similarities among the patterns shown. 2. You will facilitate the discussion by asking the following process questions: a. What shapes can you identify in the pictures? b. Do the shapes repeat in a regular pattern? c. Do you notice any gaps or overlaps? d. Based on your observations, what do you think this kind of pattern is called? 3. You will encourage learners to share their ideas and observations using complete sentences and mathematical terms whenever possible, then connect their responses to the concept that certain shapes can repeatedly cover a surface without gaps or overlaps. 4. You will divide the class into small groups and distribute cut-out polygons such as triangles, squares, pentagons, and hexagons to each group. 5. You will instruct learners to arrange the polygons on a sheet of paper and attempt to cover the surface completely without leaving gaps or overlaps while observing which polygons fit together successfully and which do not. 6. You will move around the classroom to monitor group participation, ask guiding questions, and assist learners as they explore the activity. *7.* You will facilitate a short sharing session where groups present their observations and conclusions about the polygons that tessellate successfully.
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2 **APPRECIATING LESSON RELEVANCE:** **TESSELLATIONS IN THE REAL WORLD** **Instructions:** Your teacher will present, additional real-life applications of tessellations found in Architecture, Art and Design, and Flooring and pavements. Analyze the pictures and answer the questions provided.	**Activity A.2** **APPRECIATING LESSON RELEVANCE:** **TESSELLATIONS IN THE REAL WORLD** 1. You will present real-life examples of tessellations found in architecture, art and design, flooring, and pavements, then ask learners to observe the arrangement and repetition of shapes in each picture. 2. You will facilitate the discussion by asking the following process questions: a. Where do you usually encounter these patterns?

<table>
<tr><td></td><td>

Processing Questions:
1. Where do you usually encounter these patterns?
2. Why are these shapes commonly used in tiling floors or walls?
3. What problems may occur if shapes do not fit properly?</td><td>b. Why are these shapes commonly used in tiling floors or walls?
c. What problems may occur if shapes do not fit properly?
3. You will encourage learners to explain their observations using mathematical ideas such as repetition, gaps, overlaps, and fitting shapes together.
4. You will emphasize that tessellations are practical designs used to efficiently cover surfaces without gaps or overlaps in real-life structures and artworks.
5. You will connect the lesson to the unit performance task,</td></tr>
<tr><td rowspan="4">B. Instituting New Knowledge</td><td colspan="2">B.1. Presenting Examples</td></tr>
<tr><td>Activity B.1
EXPLORING KEY CONCEPTS: TESSELLATION THROUGH ANGLES

Instructions:
2. Same group from activity 1, explore which of these polygons can tessellate the plane by repeatedly arranging the shapes.
3. After the exploration, learners answer the following:
a. Which polygons tessellate the plane?
b. Which do not?
c. What do you observe about the angles around each vertex?
d. Write a rule that determines if the polygon tessellates.</td><td>Activity B.1
EXPLORING KEY CONCEPTS: TESSELLATION THROUGH ANGLES
Begin this section by transitioning from the previous "trial and error" activity into a formal investigation of interior angles. Direct your learners to revisit the same groups and provide them with a set of regular polygons: equilateral triangles, squares, regular pentagons, and regular hexagons. Instruct them to arrange multiple copies of a single shape around a single central point, or vertex, and ask them to observe which shapes leave a gap or overlap. As you circulate, prompt them to use their protractors or prior knowledge to identify the interior angle of each shape. Guide the learners to observe that:
1. Six triangles meet around a point (60° each).
2. Four squares meet around a point (90° each).
3. Three hexagons meet around a point (120° each)

Leads the learners to conclude that the sum of the angles around a point must be exactly 360° for a tessellation to occur.</td></tr>
<tr><td colspan="2">B.2. Discussing the Concept</td></tr>
<tr><td>Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Instructions:
The concepts presented are the key ideas to be learned from the lesson, listen attentively and participate actively in the discussion.</td><td>Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

1. Define a tessellation clearly as a repetitive pattern on a two-dimensional plane that fits together perfectly with no gaps and no overlaps. Focus the discussion on the Vertex Rule:</td></tr>
</table>

Key Terms

Tessellation-a repetition of geometric figures on a two-dimensional plane that has no overlaps or gaps.

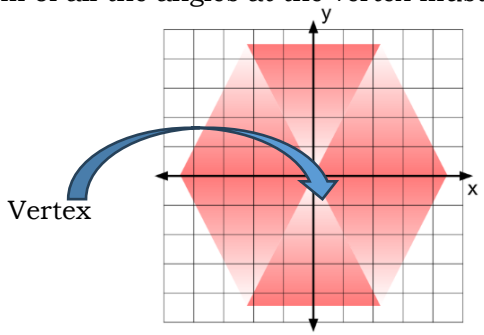
What tessellates?

When we say that a polygon tessellates the plane, we mean that:

- Copies of the polygon can be repeated endlessly
- They fit together perfectly
- They leave no empty spaces and do not overlap.

To identify whether a polygon tessellates the plane, we focus on how the polygon fits together at its vertices.

The sum of all the angles at the vertex must be equal to 360°.



Angles at the vertex

Important rule:

If the angles add up to more than 360, the shapes will overlap. If the angles add up to less than 360, gaps will appear.

Generating tessellations using geometric transformations

A tessellation is formed when a shape is repeated over a plane so that it covers the surface completely without gaps or overlaps. Instead of drawing each shape one by one, tessellations are efficiently created using geometric transformations. These transformations allow a single shape to be moved or turned in a systematic way to produce a repeating pattern.

Role of Geometric Transformations in Tessellations

emphasize that if the angles exceed 360°, the shapes will overlap; if they are less, gaps occur.

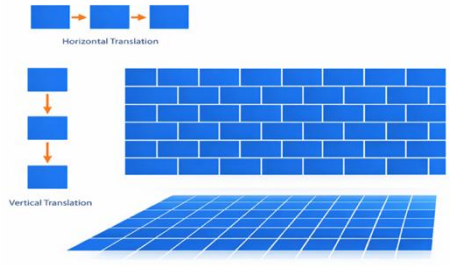
2. After defining the "what," introduce the "how" through geometric transformations. Explain that instead of drawing shapes individually, tessellations are generated through systematic movement. Use the following breakdown to guide your lecture:
   a. Translation: Describe this as "sliding" the shape without turning or flipping it. Show how a square tile on a floor is a result of horizontal and vertical translation.
   b. Rotation: Explain this as "turning" the shape around a fixed vertex. Use the equilateral triangle as your primary example, showing that since each angle is 60°, six triangles must rotate around a point to complete the 360° requirement.
   c. Reflection: Define this as "flipping" a shape to create a mirror image. Note that the staggered brick-wall pattern is generated by translation with a row-to-row offset (or, equivalently, by a glide reflection) rather than by reflection alone, since reflecting a rectangle across either axis returns the same rectangle.
3. Encourage learners to see that while simple patterns use one movement, complex and artistic designs often combine translation, rotation, and reflection.
4. The guided practice below scaffolds learners' understanding by modeling a complete tessellation example step-by-step. It explicitly shows where and how to begin, supporting learners who are not yet ready for independent construction. The activity is contextualized using Capiz shell tile floors commonly found in Filipino homes, strengthening relevance and engagement.

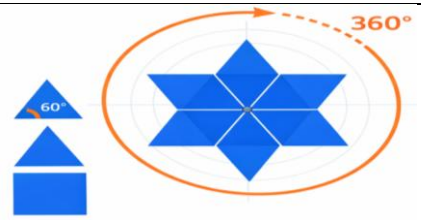
After the discussing the key ideas, provide demonstrations following the instructions below.

Guided Practice: Building a Tessellation Together

This guided practice builds learners' confidence in creating tessellations by modeling a step-by-step example using the vertex angle rule and geometric transformations, anchored in real-life patterns such as Capiz shell tile floors.

Materials

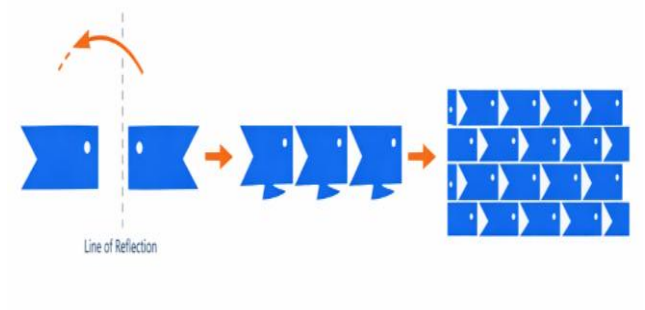
	Geometric transformations change the position or orientation of a figure without changing its size or shape. In tessellations, the most used transformations are translation, rotation, and reflection. 1. **Translation** means sliding a shape from one position to another without turning or flipping it. a. The shape keeps the same orientation. b. All points of the shape move the same distance in the same direction. **Example**: When a square is translated horizontally and vertically, copies of the square fit perfectly beside one another. This is why square tiles are commonly used on floors.  In tessellation, a polygon that tessellates the plane can be repeatedly translated to fill the plane completely. Common polygons using translation are squares, equilateral triangles, and regular hexagons. 2. **Rotation** means turning a shape around a fixed point called the center of rotation. a. The shape turns by a certain angle(e.g., 60°, 90°, 120°) b. The shape remains congruent to the original. **Example:** An equilateral triangle can be rotated around one of its vertices. Since each interior angle is 60°, six triangles can rotate around a point to form a full 360°, creating a tessellation.	• Graph paper (or projected grid) • Cut-out **regular squares** (pre-cut for consistency) • Ruler, pencil, and protractor • Projector/board (or GeoGebra for dynamic visualization) • 1–2 images of **square Capiz shell tile floors** (real-life Filipino example) **1. Teacher Modeling (I DO)** a. Show an image of square Capiz shell tiles covering a Filipino home floor with no gaps or overlaps. b. Pose the bridging question: *"What do you call a pattern of regular polygons that covers a plane completely, with no gaps or overlaps?"* **Expected response:** Tessellation (or tiling pattern). c. On the board or projected grid, draw **one regular square** with side length 2 cm. The square is chosen as the base polygon. A square has sides. d. Explain and compute its interior angle: $$\text{Interior angle} = \frac{(n-2)\times 180^\circ}{n} = \frac{(4-2)\times 180^\circ}{4} = 90^\circ$$ e. Place a second square adjacent to the first by sliding it to the right; this movement is called a **translation**, which is a slide without rotation or reflection. f. **Label directly on the drawing:** *Translation → right* **2. Guided Questioning (WE DO)** Ask learners the following aligned questions:



In tessellation, rotation is possible when the sum of the angles around a vertex is exactly 360̊.

3. Reflection means flipping a shape over a line, creating a mirror image. The reflected shape is reversed but remains the same size and shape.

Illustration



This diagram shows:

a. An original figure on one side
b. A line of reflection (dashed vertical line)
c. The reflected image on the opposite side
d. Both figures are equal in size and shape, but their orientation is reversed

In tessellation, reflection is commonly used in decorative and artistic tessellations, particularly when creating patterns with symmetrical elements.

Combining transformations

- **What regular polygon are we using?** *Expected:* Squares
- **Do the shapes repeat regularly using a transformation?** *Expected:* Yes, identical squares repeat.
- **Are there gaps or overlaps?** *Expected:* No gaps and no overlaps.
- **Does this arrangement tessellate the plane? Why?** *Expected:* Yes, because the squares fit perfectly edge-to-edge.

3. Vertex Rule Check (WE DO – with computation)

a. Focus on one vertex where four squares meet.
b. Ask: *"What is the interior angle of a regular square?"* **Expected:**
c. *Compute together:*

Or

d. Emphasize that for regular polygons to tessellate the plane, the angles around each vertex must add up to exactly , which represents one full turn.
e. Co-create the rule with learners:
Tessellation Vertex Rule: A regular polygon tessellates the plane if copies meet at a vertex such that

$$k \times \left(\frac{(n-2) \times 180^\circ}{n} \right) = 360^\circ, \quad k \geq 3.$$

4. Learners Participation (YOU DO – with guidance)

a. Learners replicate the model on graph paper.
b. With teacher guidance, learners:
   - Add **two more squares** using translation (e.g., upward and leftward).
   - **Label each movement**: *Translation → up*, *Translation → left.*
c. Teacher prompts:
   - Did you translate, rotate, or reflect the square?
   - Where do you check that the vertex angles add up to ?

5. Quick Guided Reflection

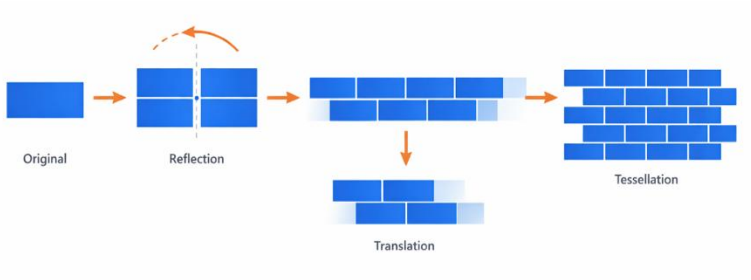
Learners complete orally or in their notebooks:

- **Polygon:** Regular squares (, interior angle)

The transformations commonly used to generate tessellations in this lesson are rigid motions: translation, rotation, and reflection. These transformations preserve size and shape, so repeated copies can fit edge-to-edge.

Example. A staggered brick-wall pattern is formed by horizontal translation within each row, combined with a horizontal offset between consecutive rows. This combination is mathematically equivalent to a glide reflection: a translation parallel to a line, combined with a reflection across that line. Since rectangles are symmetric across both axes, the same visual pattern can also be generated by translations alone with a row-to-row offset.

Illustration:



This combination creates more complex and artistic patterns, such as those found in textile designs and artworks.

- **Transformations used:** Translation only
- **Vertex check:**

Conclusion: Regular squares tessellate the plane.

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.

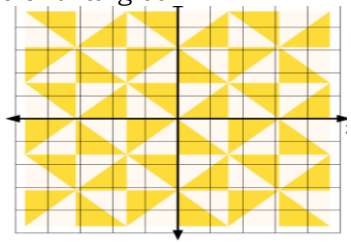
1. Refresh concepts of regular polygons, interior angles, and geometric transformations (translation, rotation, reflection). Remind them that tessellations have no gaps or overlaps.
2. Have learners draw an accurate regular polygon (equilateral triangle, square, or hexagon).
3. Instruct learners to apply at least two transformations (labeling them on the design) to expand the pattern into a grid or a cluster of 10+ shapes.
4. Have learners select a central vertex where polygons meet, list the interior angles surrounding it, and compute their sum. Emphasize that it must equal .
5. Guide learners to write precise mathematical descriptions of their moves.
6. Pair learners up to swap designs and check their partner's work for:
    a. Absence of gaps/overlaps.

<table>
<tr><td></td><td colspan="2">b. Accurate angle-sum computation.
c. Whether the interior angle is a factor of .
7. Have learners offer constructive feedback and allow time for revisions.

Processing Questions:
1. Facilitate a brief class discussion using these key questions:
2. Why do only certain regular polygons tessellate a plane?
3. How did transformations help you create your design?
4. Why is the angle sum around a vertex critical in tessellations?
5. How are tessellations applied in real-world architecture, art, or design</td></tr>
<tr><td rowspan="2">C. Demonstrating Knowledge and Skills</td><td colspan="2"></td></tr>
<tr><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: APPRECIATING TESSELLATION IN REAL LIFE

Instructions:
1. Observe the displayed images of floor tiles, brick walls, artworks, and traditional textile patterns such as binakol or indigenous weaves. Focus on how the shapes meet without gaps or overlaps.
2. Examine the “seams” or “joints” carefully and identify the shapes used in the patterns.
3. Answer the guide questions and share your observations with your group or classmates. Think about where tessellations can be seen in everyday life and why these patterns are useful.
4. Discuss how mathematics helps create these patterns, especially how shapes fit perfectly around a point to form a complete design.
5. Reflect on how geometric transformations such as translation, rotation, and reflection are used to fill spaces completely and create visually appealing patterns.
6. Use your observations and ideas as inspiration for your own artwork or contribution to the class exhibit on symmetry and tessellation.</td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: APPRECIATING TESSELLATION IN REAL LIFE

1. Visual Gallery Walk:(This activity may be implemented using locally available and contextually appropriate materials.)
a. Display high-quality images of floor tiles, brick walls, M.C. Escher’s artworks, and traditional textile patterns (such as binakol or indigenous weaves).
b. Direct learners to look specifically at the "seams" or "joints" where shapes meet. This reinforces the core definition of a tessellation: a repetition of figures with no overlaps and no gaps.
2. Facilitating the Discussion: o Use the guide questions to move from observation to critical thinking.
a. Where do we see these? Encourage learners to look beyond the classroom—mentioning honeycombs, sidewalk pavements, or even the patterns on a soccer ball.
b. Why are they useful? Guide the conversation toward efficiency and cost. Explain that in construction, shapes that tessellate perfectly (like squares or hexagons) minimize wasted material and provide structural integrity.
c. How does math help? Connect this back to the Vertex Rule. Explain that "beauty" in these patterns often comes from the mathematical precision of the sum of angles equaling exactly 360°.
3. Conclude by explaining that whether it is a simple bathroom tile or a complex piece of art, the underlying math remains the</td></tr>
</table>

		same: it is the systematic movement of a shape—through translation, rotation, or reflection—to fill a space completely.
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Option I: Core Task** **Instructions:** Accomplish the indicated tasks. **Part I.** Using what you learned from the activities and discussion, complete the statements below. Be ready to justify your answers with an example. 1. A tessellation is a mathematical pattern that _____. 2. A polygon tessellates the plane if _____. 3. The Vertex Rule is important in tessellations because _____. 4. Regular polygons that can tessellate the plane include _____. 5. Tessellations are formed through geometric transformations such as _____. 6. Tessellations are applied in real-life contexts like _____. **Part II**. Compare answers with a partner, then the class consolidates a shared generalization guided by the teacher. **Option 2. Guided Support** **Instructions:** Accomplish the indicated tasks. **Part I. Picture–Concept Matching** Match Column A with Column B**.** **Column A (Pictures):** 1. Tessellation made of triangles  2. Tessellation made of squares 3. Pattern with gaps	**Activity C.2. WRAPPING UP THE LESSON** **Option 1** 1. Present the incomplete statements to learners and explain that they must complete each one using their prior learning. 2. Remind learners to think of examples from earlier tessellation activities when answering. 3. Encourage individual work first to promote independent thinking. 4. Guide learners to pair up and compare answers after completion. 5. Facilitate a class synthesis by highlighting correct ideas and clarifying misconceptions. 6. Emphasize justification by asking: *"What example supports your answer?"* **Option 2.** **Part I** 1. Show each image clearly and allow learners time to observe. 2. Ask guiding questions such as: a. "Do the shapes leave gaps?" b. "What transformation is used here?" c. "Where do the shapes meet?" 3. Assist learners in matching Column A with Column B through discussion. 4. Reinforce correct reasoning rather than giving direct answers immediately. **Part II.** 1. Introduce the word bank and review key terms briefly. 2. Read each sentence aloud and pause for learner input. 3. Provide scaffolding prompts: • "What is missing in a tessellation pattern?" • "What must the angles around a vertex equal?" • "What transformations did we use earlier?" 4. Allow collaborative answering in pairs or small groups. 5. End with a quick check for understanding.

4. Pattern formed by sliding shapes

Column B:

A. Shapes fit without gaps because angles around a point add to 360°

B. Tessellation formed using translation

C. Not a tessellation (has gaps or overlaps)

D. Example of a polygon that tessellates the plane

Part II. Guided Sentence Completion

Complete each sentence using the words from the **Word Bank.**

Word bank			
Tessellation	360°	translation	rotation
Reflection	gaps	overlaps	polygon

1. A _____ is a pattern that covers a plane with no _____ or _____.
2. A _____ tessellates the plane when the angles around a vertex add up to _____.
3. Tessellations can be formed using geometric transformations such as _____, _____, and _____. whether it tessellates. Complete: "The claim is __ because __."

C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)

Activity C.3. ASSESSING LEARNING OUTCOMES

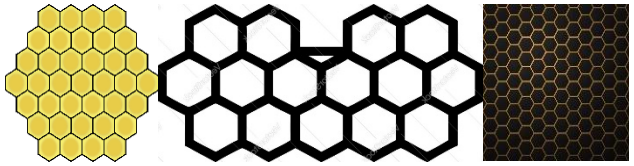
Before facilitating this activity, ensure that all learners have a copy of the LAS.

1. Begin by reviewing the key concepts of tessellations, the vertex rule, and geometric transformations to activate prior knowledge. Clearly explain the three parts of the activity and ensure learners understand that explanations and justifications are required, not just final answers.
2. Guide learners as they work through each task, moving from basic identification in Part I, to pattern analysis in Part II, and finally to coordinate geometry application in Part III. Encourage learners to explain their reasoning using mathematical concepts such as angle measures, transformations, and properties of polygons.
3. Provide support by asking guiding questions that prompt deeper thinking, especially when learners struggle to justify answers or visualize transformations. Monitor learners' work for accuracy in identifying tessellating shapes, describing transformations, and correctly plotting and labeling points in the coordinate plane.
4. Conclude by reinforcing that tessellations are governed by mathematical rules that ensure no gaps or overlaps, and that geometric transformations are key tools in creating repeating patterns seen in both mathematical contexts and real-life designs.

C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)

	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Begin by reviewing the Vertex Rule and reminding learners that a tessellation occurs when angles meeting at a point total exactly 360°. Emphasize that learners must show computations and explain their reasoning, not just provide answers. 2. Guide learners in the remediation tasks by helping them connect angle measures to the number of polygons that can meet at a vertex and identifying why some shapes do not tessellate. Encourage clear explanations, especially when interpreting angle sums and determining validity of tessellations. 3. For enrichment activities, allow learners to explore creatively while still applying mathematical rules. Support them in recognizing how transformations such as translation and rotation help generate patterns, even in irregular or combined polygon tessellations. Encourage observation, experimentation, and discussion of how the Vertex Rule still applies in more complex designs. 4. Monitor learners' work to ensure mathematical accuracy while also valuing creativity and exploration. Conclude by reinforcing that tessellations can be both structured and creative, but all patterns must still follow the fundamental rule of 360° around a vertex.

III. CONTENT	**Lesson 1.8. ESCHER-TYPE TESSELLATION**	
IV. OBJECTIVES	**At the end of the lesson, learners are will be able to:** **1. define an Escher-type tessellation as a repeating artistic tessellation made by modifying a tessellating base tile into a figurative shape that interlocks without gaps or overlaps;** **2. construct an Escher-type tessellation from a square, equilateral triangle, or regular hexagon by using a cut-and-attach process that preserves area and creates matching edges;** **3. label the base tile, modified tile, cut edge, attached edge, and at least four interlocking copies of the modified tile; and** **4. write a short mathematical justification explaining why the modified tile still tessellates, using the ideas of area preservation and edge-matching.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1.*** **Eliciting *Prior Knowledge***	
	Activity A.1 **LEVELING LEARNER READINESS:PATTERN HUNT** **Instructions:** 1. Observe the images of honeycombs, bathroom tiles, and brick walls. 2. Identify and discuss what is common among these patterns. 3. Study the concept of **regular tessellations**—covering a flat surface using geometric shapes without gaps or overlaps. 4. Examine the given shapes. 5. Determine which regular polygons (triangles, squares, pentagons, or hexagons) fit together perfectly to form a repeating pattern.	**Activity A.1** **LEVELING LEARNER READINESS:PATTERN HUNT** 1. Present images (honeycomb, tiles, brick wall). *Include these visuals in the LE for easy reference.* Learners observe and respond to guide questions arranged from simple to abstract: • What shapes do you see? • What is common among the patterns? • Do the shapes leave gaps or overlap? • Where do you see these patterns in real life? • Can all shapes tessellate? Why or why not? 2. Organize responses (repeating shapes, no gaps, organized patterns) and introduces tessellation as patterns that cover a

<table>
<tr><td></td><td></td><td>surface without gaps or overlaps. Learners then examine polygons and answer: "Which shapes can tessellate? Why?"
3. Clarify that triangles, squares, and hexagons tessellate (angles form 360°). Regular pentagons do not tessellate by themselves because 108° does not divide 360°. Some non-regular pentagons can tessellate, but they are not the focus of this activity.
4. Learners identify examples in the classroom, reinforcing real-life application.</td></tr>
<tr><td></td><td colspan="2">A.2. Establishing the Purpose of the Lesson</td></tr>
<tr><td></td><td>Activity A.2.
APPRECIATING LESSON RELEVANCE: OBSERVING AND ANALYZING

Instructions:
1. Observe the artworks of M.C. Escher such as Sky and Water I or Reptiles.
2. Analyze how simple geometric shapes (e.g., squares) are transformed into recognizable figures like birds, fish, or lizards.
3. Discuss with your classmates your ideas on how these transformations were made while still allowing the shapes to fit together without gaps or overlaps.
4. Understand that tessellation involves modifying and reorganizing geometric figures to create repeating patterns that can extend infinitely.
5. Pay attention to how Escher preserves perfect fitting while changing the forms of the shapes.
6. Relate this to your role in the Math-Art Exhibit: The Hidden Geometry of Design, where your group will create four artworks.
7. As a Tessellation Specialist, you will design a tessellation artwork using either regular tessellation methods or Escher-inspired transformations, based on today’s lesson.
8. Prepare to include a short description identifying the polygons and transformations used in your artwork.</td><td>Activity A.2.
APPRECIATING LESSON RELEVANCE: OBSERVING AND ANALYZING

1. Presents selected artworks of M.C. Escher (e.g., Sky and Water I, Reptiles).
Note: Include sample images, diagrams, or illustrations directly in the Learning Episode (LE). If possible, provide links or a short video presentation as an alternative for easier teacher access.
2. Learners observe the artworks and respond to guide questions (from simple to abstract):
a. What shapes or patterns do you see repeating?
b. What figures (animals/objects) can you recognize?
c. How are the shapes arranged without gaps or overlaps?
d. How did the artist transform simple shapes into complex figures? Why do the shapes still fit together perfectly?
3. Learners share their ideas while the teacher facilitates and organizes responses, highlighting key concepts such as transformation, repetition, and continuous patterns.
4. Emphasize that Escher “breaks and reforms” simple shapes while maintaining a perfect fit, introducing the idea of Escher-type tessellations.</td></tr>
<tr><td></td><td colspan="2">B.1. Presenting Examples</td></tr>
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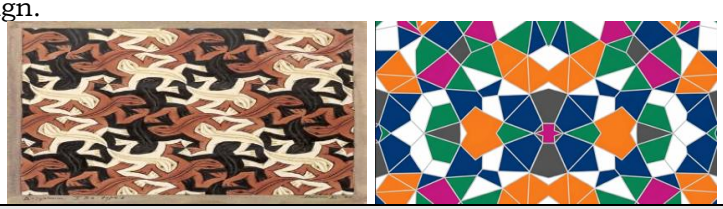
<tr><td rowspan="3">B. Instituting New Knowledge</td><td>Activity B.1. EXPLORING KEY CONCEPTS

Instructions:
1. Study the concept of Escher-type tessellations, where a basic shape (such as a square or hexagon) is modified to create a repeating pattern.
2. Observe how M.C. Escher transformed simple geometric shapes into artistic figures like birds, fish, or reptiles that fit together without gaps or overlaps.
3. Identify how these shapes repeat continuously across a surface to form a seamless design.
4. Create your own Escher-type tessellation by modifying a base shape.
5. Cut, slide, or reshape parts of the shape while keeping its edges consistent.
6. Apply geometric transformations such as translation (sliding) and rotation (turning) to transfer the modified parts to other sides of the shape.
7. Ensure that all edges still match exactly so the shape can repeat perfectly.
8. Produce a continuous and visually appealing tessellation design.
</td><td>Activity B.1. EXPLORING KEY CONCEPTS

1. Begin by recalling prior knowledge on tessellations and Escher’s artworks. Learners briefly share observations about repeating patterns without gaps or overlaps.
2. Introduce Escher-type tessellations as patterns formed by modifying a base shape (e.g., square or hexagon) into artistic figures while still fitting perfectly.
3. Note: Include sample pictures, diagrams, or illustrations of Escher tessellations in the LE. If possible, add a short video or visual demonstration to help learners better visualize the concept.
4. Demonstrate how a shape is modified using translation (slide) and rotation (turn) while keeping edges matching.
5. Synthesize responses and emphasize that even if shapes become artistic, matching edges are essential for a continuous tessellation.
6. Highlights real-life applications in art, design, and digital media, then prepares learners for creating their own tessellations in the next activity.

Processing Questions:
1. What is a tessellation?
2. What do you notice about Escher’s patterns?
3. How are the shapes changed from simple polygons?
4. What transformations are used to modify the shapes?
5. Why must the edges still match after modification?
6. What happens if edges do not match?</td></tr>
<tr><td colspan="2">B.2. Discussing the Concept</td></tr>
<tr><td>Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Instructions:
1. Use a square, equilateral triangle, or regular hexagon as your base tile. For the first demonstration, use a square ABCD.
2. Draw and label the original square ABCD. Mark the top edge AB and the bottom edge DC.
3. Cut one curved or angular piece from edge AB. Label the removed piece Cut Piece 1 and label the remaining part on AB as the cut edge.</td><td>Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

1. The Introduce the “Nibble Technique” as a method for creating a tessellating shape from a simple square. A short instructional video on Escher tessellation construction may be shown to support understanding (optional resource from reliable educational platforms).
2. Prepare a 3×3 inch cardstock square and show it to learners as the base shape.
3. Explain that the shape will be modified using translation (sliding without turning or flipping).</td></tr>
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<table>
<tr><td></td><td>4. Attach the same Cut Piece 1 to edge DC in the same horizontal position. Label the new part on DC as the attached edge. Do not discard the cut piece.
5. Trace the modified tile four times to form a 2-by-2 block. Make sure the cut edge of one tile matches the attached edge of the adjacent tile.
6. Complete: (a) The area of the modified tile is the same as the area of the original square because ___. (b) Adjacent copies fit without gaps because ___. (c) If the cut piece were discarded instead of attached, ___.

Processing Questions:
1. What happens to the shape when we cut and move parts of it?
2. Why must the cut-out pieces keep the same orientation?
3. Why can the modified shape still tessellate?
4. How does this process relate to Escher's artwork?</td><td>Step 1. Left–Right Modification
• Cut a small "nibble" from the left side of the square.
• Slide the cut-out piece to the right side.
• Attach it while maintaining the same orientation.
Step 2. Top–Bottom Modification
• Cut another small shape from the top edge.
• Slide it directly to the bottom edge.
• Attach it without rotating or flipping.
Step 3. Show the final modified shape and explain that it now looks irregular or "funky," but still fits perfectly with copies of itself.</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3:
PRACTICING LEARNED SKILLS: APPLYING THE NIBBLE TECHNIQUE

Before facilitating this activity, ensure that all learners have a copy of the LAS.
1. The teacher introduces the activity as independent practice of the previously demonstrated Nibble Technique.
2. Learners use a square piece of scrap paper to create their own tessellating template.
3. Start with a square paper as the base shape.
4. Cut a small "nibble" from one side.
5. Slide the cut-out piece to the opposite side and attach it securely.
6. Repeat the process for another pair of opposite sides.
7. Ensure that pieces are only slid (translation), not flipped or rotated.
8. Trace the modified shape repeatedly on paper to check if it forms a continuous tessellation without gaps or overlaps.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: ESCHER ARTIST
Materials: Paper, tessellation template, pencil, coloring materials

Instructions:
1. Create a draft of your Escher Artist output for the class Math-Art Exhibit.
2. Your draft must show on one sheet:
Base tile</td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: ESCHER ARTIST
1. Begin by reminding learners of the key idea: shapes must fit together without gaps or overlaps.
As they work:
2. Guide learners through each step and ensure they understand tessellation rules.
3. Model how to trace and align shapes correctly at the start.
4. Walk around the classroom to check for gaps, overlaps, and progress.</td></tr>
</table>

	• Draw and label the original square, equilateral triangle, or regular hexagon used as the starting tile. **Modified tile** • Show how the tile was changed by cutting and attaching pieces; label the cut edge and attached edge. **Four copies** • Trace at least four interlocking copies of the modified tile in a 2-by-2 block or a connected row-and-column arrangement. **Artistic details** • Add simple details that turn the tile into a figure such as a fish, bird, or lizard. **Mathematical justification (4 to 5 sentences)** Address each of the following: a. What base tile did you use? b. What transformation or movement was used to attach the cut piece? c. Why was the total area preserved? d. How do the matching edges prevent gaps and overlaps? How can this work connect to the common theme of your group exhibit	5. Provide encouragement and assist learners who are struggling. 6. Give constructive suggestions to improve accuracy and creativity. 7. Facilitate the processing questions**:** a. What figure or object does your tessellation resemble? b. How did you decide on the design or character for your artwork? c. What challenges did you experience while ensuring your shapes fit perfectly? Note: d. Tessellations must remain continuous and gap-free. e. Mathematical structure and artistic creativity must both be present. f. Shapes can be transformed into meaningful images while preserving pattern integrity.
	C.2. Making Generalization	
	Activity C.2. **WRAPPING UP THE LESSON: THE GALLERY WALK** 1. Display your tessellation artwork from the **Making Real-World Connections** activity on the wall for everyone to observe. 2. Walk around the classroom and examine the tessellation designs created by your classmates. 3. Observe your own modified shape and compare it with the original square. 4. Reflect on this question: **How does the perimeter of the modified shape compare with the perimeter of the original square?** 5. Notice that the perimeter increased while the area remained the same. 6. Discuss with a classmate why this happened and explain how the shape changed while still fitting together perfectly without gaps or overlaps.	**Activity C.2.** **WRAPPING UP THE LESSON: THE GALLERY WALK** 1. Conduct a Gallery Walk where learners display and observe tessellation artworks. 2. Learners provide peer evaluation using a simple rubric. 3. Facilitate the reflection and discussion. Criteria / Excellent (3) / Good (2) / Needs Improvement (1) Accuracy of Tessellation / No gaps or overlaps; perfect fit / Minor gaps or alignment issues / Many gaps/overlaps Creativity of Design / Highly creative and original transformation / Some creativity shown / Minimal or unclear transformation Visual Presentation / Neat, clear, and well-colored / Acceptable presentation / Unfinished or unclear output

	Processing Questions: 1. What similarities do you notice among the tessellations? 2. How did repeated shapes form a continuous pattern? 3. How did your shape change from the original square? 4. What happened to the perimeter and area of your shape?	Note: • Tessellations must be continuous and gap-free. • Art and mathematics are combined through transformation and creativity. • Learning is enhanced through sharing and peer feedback.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. **ASSESSING LEARNING OUTCOMES: EXIT TASK** Before facilitating this activity, ensure that all learners have a **copy** of the LAS. **1. Draw the Original Figure** • Ask learners to draw square **ABCD** and sketch the curved cut on side **AB**. • Check that the figure and labels are correct. **2. Construct Tile T** • Guide learners in translating the cut piece to side **DC** without rotating or reflecting it. • Remind them to label the cut edge and attached edge. **3. Create the Tessellation** • Instruct learners to draw four copies of Tile T in a **2 × 2 arrangement**. • Check for gaps or overlaps in the pattern. **4. Explain the Mathematics** • Ask learners to explain why Tile T tessellates. • Encourage them to use the terms **area preservation** and **edge-matching**. **5. Reflect and Analyze** • Ask learners what would happen if the cut piece were discarded instead of attached. • Synthesize responses by emphasizing the importance of matching edges in tessellations.	
	C.4. Additional Activities	
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a **copy** of the LAS. **1. Introduce the Task** • Present the rubric and explain the activity requirements. • Review the concept of rotation and show a sample Escher-inspired tessellation. **2. Guide the Tile Construction** • Ask learners to draw a square or equilateral triangle, cut a small shape from one side, rotate it 180°, and reattach it. • Check that the rotated piece is attached correctly. **3. Monitor Tessellation Creation** • Instruct learners to trace and rotate the modified tile repeatedly to create a tessellation. • Ensure that the pattern covers the plane without gaps or overlaps. **4. Encourage Creativity**	

	• Ask learners to add details, designs, and colors to transform the tessellation into an artistic pattern. • Remind them to maintain the repeating structure of the tessellation. **5. Facilitate Reflection** • Direct learners to write a 3–5 sentence Artist Statement explaining how rotation was used to create their tessellation. • Invite selected learners to share their designs and explanations with the class.

III. CONTENT	**Lesson 1.9. FRIEZE PATTERNS**	
IV. OBJECTIVES	**At the end of the lesson, learners should be able to:** **1. define a Frieze pattern as a strip pattern that has translation symmetry along one direction;** **2. identify the motif or generating unit and state the smallest translation vector that maps a given Frieze pattern onto itself;** **3. identify additional symmetries in a Frieze pattern, such as vertical reflection, horizontal reflection, glide reflection, and half-turn rotation;** **4. justify whether a given strip is or is not a Frieze pattern by checking for congruent motifs repeated at equal distances; and** **5. create a Frieze pattern strip with at least four motif repetitions and write a short mathematical justification of the transformations used.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITTAION GUIDE)**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge***	
	Activity A.1. **LEVELING LEARNER READINESS: READY, SET, TRANSFORM** **Instructions**: Follow the steps below to master the geometric "moves" that create beautiful Frieze patterns. 1. Observe: Walk through the gallery (or view the slides) showing the four basic transformations. Look closely at the Translation, Reflection, Rotation (180°), and Glide Reflection examples. 2. Analyze: For each image, determine exactly how the shape moved from its original position to its new one. Did it slide, flip, turn, or do a combination of two moves? 3. Identify: Participate in the “Match the Move” mini-quiz. Look at the footprint patterns provided and name the specific transformation that describes how the "foot" moved from step to step. 4. Explain: Discuss your findings with a partner. Use mathematical vocabulary to justify why a specific move is a "Reflection" rather than a "Glide Reflection." **Processing Questions**	**Activity A.1.** **LEVELING LEARNER READINESS: READY, SET, TRANSFORM** 1. You will formally introduce the lesson, explaining that geometric transformations are the essential "building blocks" used to create the linear repeating designs known as Frieze patterns. 2. You will display a slide presentation (or lead a guided Gallery Walk) featuring distinct, labeled visual examples of the four basic transformations: Translation Reflection Rotation (180°) Glide Reflection 3. For each transformation example, the teacher will pose targeted observation questions, such as: "What is happening to this shape? Has it flipped, turned, or simply slid?" "How did the shape's orientation change?" "Where is the line of reflection/center of rotation in this example?"

	Use these questions to guide your thinking during the activity*:* 1. How can you tell the difference between a simple Reflection and a Glide Reflection? What is the "extra step" involved in a glide? 2. When a shape undergoes a Translation, does its size, shape, or orientation change? Why is this important for creating a consistent pattern? 3. If you rotate a footprint 180°, will it look the same as if you had simply reflected it across a horizontal line? Why or why not? 4. Look around the room or think of your home. Where have you seen these "moves" used in borders, clothing, or floor tiles? •	4. You will actively facilitate discussion, encouraging learners to describe the transformation in their own words before reinforcing the correct mathematical terminology. 5. To formatively assess learner readiness, the teacher will present the "Match the Move" mini-quiz. The teacher will show an image of a footprint moving in different ways and require learners to identify the corresponding transformation: Translation: A footprint sliding along a line. Reflection: A footprint flipped across a line. Rotation: A footprint turned around a point. Glide Reflection: A footprint both moved and mirrored along a line. 6. During the activity, you will circulate, observing responses to gauge individual and class-wide understanding. The teacher will immediately clarify any misconceptions (e.g., confusing reflection with glide reflection) and provide corrective feedback. The activity concludes once the teacher determines that learners can accurately recognize and name all four transformations, setting the foundation for the next stage of the lesson.
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. **APPRECIATING LESSON RELEVANCE: INFINITE PATTERNS** In this activity, we are going to explore patterns all around us! Look at the photos of local architecture, such as wrought iron fences, ethnic weaving, or Greek key designs. Pay close attention to how the designs repeat. Now think: *"If you had to describe this pattern to a robot so it could print it forever, what would be the shortest set of Instructions: you could give?"* This helps us understand the idea of a generating unit, which is the smallest section of a pattern that can be repeated to create the full design. Discuss your ideas with your partner and see if you can figure out what part of the pattern repeats to make the whole design. In the unit Math-Art Exhibit titled "The Hidden Geometry of Design," your curatorial team will produce four artworks. As Frieze Specialist (or as a contributor to the Frieze Specialist's work), you will produce a Frieze pattern artwork that demonstrates one of the seven Frieze types, accompanied by a description of the generating unit and the	**Activity A.2.** **APPRECIATING LESSON RELEVANCE: INFINITE PATTERNS** 1. Present a curated "Cultural Pattern Gallery" (e.g., ethnic weaving like *Inabel* or *T'nalak*, wrought iron fence designs, or Greek key motifs). 2. Direct learners to focus on the linear nature of these designs. Ask: *"How does the design 'travel' from one side of the fabric or fence to the other?"* 3. Pose the following scenario to the class: *"Imagine you are programming a printing robot. To save memory, you can only give it one small shape and a set of transformational rules. If the robot needs to print this pattern forever, what is the smallest piece it needs to start with?"* 4. Introduce the term Generating Unit as the "minimum instruction" the robot needs. 5. Ask learners to examine the provided photos to find the smallest repeating part (the generating unit).

<table>
<tr><td></td><td>transformations used. Today's lesson directly prepares you for that role.
 </td><td>6. Tell learners to name the specific transformations (Translation, Reflection, Rotation, or Glide Reflection) required to move that unit along the line.
7. Learners justify their choices
8. Circulate through the room while learners are analyzing the designs.
9. For learners struggling to see the unit, use a "viewfinder" (a small paper cutout) to help them isolate one part of the pattern. Ask: "If I cover everything but this one square, can you still recreate the whole line using the moves we learned?"

Processing Questions:
1. In the weaving patterns you observed, which transformation is most common? Why do you think weavers choose that specific move?
2. Why is finding the "shortest set of Instructions:" (the generating unit) better than describing every single shape in the pattern individually?
3. Does a pattern look different to you now that you know it follows a strict mathematical rule?
4. If you were a digital textile designer, how would understanding the "generating unit" help you create a large-scale wallpaper design?</td></tr>
<tr><td rowspan="2">B. Instituting New Knowledge</td><td colspan="2">B.1. Presenting Examples</td></tr>
<tr><td>Activity B.1.
EXLPORING THE CONCEPTS:
WHAT MAKES A PATTERN A FRIEZE PATTERN

Instructions:
1. Study three strip patterns shown by your teacher.
Strip A. the same flower motif repeats every 4 cm along a horizontal strip; all flowers have the same size and orientation.
Strip B. a flower motif is repeated along a horizontal strip, but each flower is twice as large as the previous one.
Strip C. a triangle motif is turned 180 degrees, but the distances between consecutive motifs are not equal.

2. For each strip, answer:
a. Is the motif congruent each time?</td><td>Activity B.1.
EXLPORING THE CONCEPTS:
WHAT MAKES A PATTERN A FRIEZE PATTERN

1. Formal Definition and Connection:
a. Bridge the "Robot" activity from A.2 to a formal mathematical definition. Write on the board: "A Frieze Pattern is a design that repeats in one direction (linearly) using translational symmetry."
b. Explain that every Frieze pattern must have translation, but it can be "spiced up" with reflections, rotations, or glides.
2. Deconstructing the "Generating Unit":
a. Use a clear visual aid of a simple Frieze. Use a physical or digital "frame" to isolate the Generating Unit.
b. Show how this one unit, when subjected to a specific "rule" (transformation), creates the entire infinite line.</td></tr>
</table>

	b. Is there a fixed slide or translation that maps the whole strip onto itself? c. Is the strip a Frieze pattern? Why or why not? d. What is the smallest translation vector, if one exists? 3. Complete in your own words: A Frieze pattern is _____. **Processing Questions:** 1. Why is **translation** the only transformation that *must* be present in every Frieze pattern? What would happen if we took it away? 2. If you see a pattern where the shapes are "upside down" every other step, which transformation is likely being used? 3. Why might a cultural designer choose a **rotation** move instead of a simple **translation**? How does it change the way your eye moves across the design?	c. Point out how an architect or a weaver uses this mathematical "rule" to ensure a border remains consistent and aesthetically pleasing. 3. Visual Analysis of Transformation Combinations (Indicator 2): a. Instead of deep-diving into the complex math of all seven types, present three distinct "Transformation Recipes": • Recipe A: Translation only (The "Hop"). • Recipe B: Translation + Vertical Reflection (The "Sidestep"). • Recipe C: Translation + 180° Rotation (The "Spin"). b. Guide learners to see how adding just one extra transformation rule completely changes the "vibe" and structure of the art. 4. Pose targeted questions to the class while displaying different patterns: a. *"If I slide this unit but do not flip it, does it match the next part of the pattern?"* b. *"Where is the 'mirror line' in this weaving design? Is it horizontal or vertical?"* c. *"How many different moves is the 'robot' performing here?"* 5. Preparation for Application (Indicator 5): a. The Teacher's Role: Provide a "Transformation Toolkit" (a small reference card) listing the four moves. b. Challenge learners to look at a new, unlabelled pattern and "detect" the hidden moves. This prepares them for the independent classification and creation tasks in the next lesson phase.
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Instructions**: The following are the key ideas to be learned. Listen attentively and participate actively. **1. Frieze Pattern** A frieze pattern is a geometric design that repeats infinitely in two opposite directions along a single straight line (a strip). It is created by applying repeated geometric transformations to a base design. **2. Motif (Generating Unit)**	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** Because frieze patterns require spatial reasoning, avoid relying on textbook definitions alone. Use an inductive, discovery-based strategy. Introduce a singular, tangible "motif," manipulate it physically or digitally, and guide LEARNERs to notice the geometric rules themselves before introducing formal vocabulary. **1. Hook and Discovery** **Goal:** Establish the concept of a motif and translation dynamically.

	The motif is the smallest, non-repeating unit of a frieze pattern. Repeating this single motif through specific geometric rules produces the entire infinite pattern. **3. Translation** Translation is the act of sliding a figure a fixed distance along a straight line without turning or flipping it. It is the foundational transformation present in *all* frieze patterns. **4. Translation Vector** A translation vector is an arrow that indicates the exact distance and direction a motif must slide to land perfectly on the next identical motif in the sequence. **5. Reflection Symmetry** A reflection occurs when a figure is flipped across a line, creating a mirror image. Frieze patterns can feature two types of reflection: a. Vertical Reflection: The mirror line is perpendicular to the direction of the strip. b. Horizontal Reflection: The mirror line is parallel to the direction of the strip, cutting it in half lengthwise. **6. Half-Turn Rotation (Rotation)** A half-turn rotation occurs when a motif is spun around a fixed point (the center of rotation) and fits perfectly back into the pattern. This means the pattern looks exactly the same right-side up and upside down. **7. Glide Reflection** A glide reflection is a composite transformation consisting of two steps executed together: a. A reflection across the central horizontal axis line. b. A translation (slide) along that same line. *Note: Unlike a simple reflection, the final shape is both flipped and shifted forward.*	**Action:** Project an image of a striking, culturally relevant border pattern (e.g., an ancient Greek key or an indigenous textile design). Distribute small, clear plastic squares to your learners, each marked with a single asymmetrical shape (like a flag or a footprint) drawn in permanent marker. **What You Say:** "Look at the beautiful border on the screen. It looks complex, but it's built from just one repeating block. Look at the plastic square in front of you. That single shape is your motif—the building block of everything we will do today. Place your square on your desk. Without spinning it or flipping it over, slide it repeatedly to the right. Mark down where it lands each time. What you are doing right now is a translation. The exact distance and direction of your slide is guided by an invisible arrow we call a **translation vector**. You've just created your first infinite pattern!" **2. Demystifying the 4 Symmetries** **Goal:** Actively demonstrate the differences between reflections, rotations, and glide reflections. **Action:** Use physical mirrors or dynamic geometry software (like GeoGebra) on your screen to show transformations live. **Addressing the Trickiest Concept (Glide Reflection):** LEARNERs frequently confuse reflection with glide reflection. Use a concrete analogy to break this down. **What You Say:** "Let's look at **Vertical vs. Horizontal Reflection**. If I place a mirror vertically across our pattern, the left side looks into the mirror and sees the right side. If I place it horizontally, the top half sees the bottom half. Now, look at **Glide Reflection**. Imagine walking on wet sand. Look at your footprints. Are they simple reflections? No, because your left foot is slightly ahead of your right foot. To get from your left print to your right print, you must do two things: flip across the center line *and* slide forward. That is a glide reflection. If you only flip without sliding, you are just jumping in place!"

8. Symmetry in Frieze Patterns

The mathematical classification of a frieze pattern depends entirely on which combinations of the four symmetries (translation, reflection, rotation, and glide reflection) map the pattern perfectly onto itself.

9. The Seven Frieze Groups

Mathematically, every frieze pattern in existence falls into one of exactly seven distinct symmetry groups, regardless of how complex the artistic design looks:

Frieze Type (Nickname)	Standard Notation	Symmetries Present
Hop	*p1*	Translation only
Step	*p11g*	Translation and Glide Reflection
Sidle	*p1m1*	Translation and Vertical Reflection
Jump	*p11m*	Translation and Horizontal Reflection
Spinning Hop	*p2*	Translation and Rotation
Spinning Sidle	*p2mg*	Translation, Vertical Reflection, Glide Reflection, and Rotation
Spinning Jump	*p2mm*	Translation, Vertical Reflection, Horizontal Reflection, Glide Reflection, and Rotation

10. Real-World Applications

Frieze patterns are globally universal and can be observed in:

a. Architectural borders, cornices, and facades
b. Decorative floor, wall, and walkway tiling
c. Textile, ribbon, and fabric border designs
d. Cultural artifacts, pottery borders, and indigenous weaves

3. The Seven Frieze Groups Challenge

Goal: Teach classification through inquiry and collaborative problem-solving.

Teacher Action: Group LEARNERs into teams of 3 or 4. Hand out a "Symmetry Cheat Sheet" showing the 7 nicknames (Hop, Step, Sidle, etc.). Give each group images of real-world patterns (e.g., iron railings, tire tracks, wallpaper borders).

The Task: Teams must analyze their images using a systematic elimination checklist to determine which of the seven groups the pattern belongs to.

Step-by-Step Learner Elimination Checklist

1. Look for reflection. Is there a horizontal mirror line? A vertical one? Or both?
2. Spin the image completely upside down (). Does it look exactly the same? If yes, it has rotation symmetry.
3. Look for the footprint effect. Is there a shifted flip? (Glide reflection).
4. Match your checklist results to the Frieze Group table to identify its group name.

What You Say: "Do not guess the group name based on looks! Be mathematical detectives. Take your pattern card. Spin it upside down on your desk. Does it look identical? If it does, you've officially detected a half-turn rotation. Cross out any group on your chart that doesn't include rotation, and keep investigating!"

4. Assessment and Closure

Goal: Consolidate learning and link geometry to human culture.

Action: Highlight how completely isolated cultures throughout history—from ancient Rome to the Navajo Nation—all independently discovered these exact same seven mathematical combinations.

<table>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3.
PRACTICING LEARNED SKILLS: GLIDE REFLECTION AND CREATION WORKSHOP
Before facilitating this activity, ensure that all learners have a copy of the LAS.
1. Draw a large, completely asymmetrical shape (such as a comma or a flag) on the chalkboard. Demonstrate the difference between the incorrect and correct ways of creating a frieze pattern to establish the concept of the generating unit.
2. Say:
“Watch the comma on the board. If I simply slide it to the right, I create a Hop pattern. But look at what happens when I flip it upside down across this middle line and then slide it. It begins to look like a series of footprints.
3. Today, you will analyze how a simple letter L moves using this ‘flip-and-slide’ mechanic, and then you will engineer your own custom border pattern.”
4. Move around the room while groups work through the five questions. Watch for learners who mistakenly think that the translation vector for the L is 3 cm.
5. Remind learners:
“Look closely at your 3 cm mark. Is that L facing exactly the same way as the original L? No, it is upside down!
6. Remember our definition of a translation vector: it must slide a motif onto an identical motif. To get an L that is upright again, you must complete a second step. If one step is 3 cm, how many centimeters do you need to travel to find its identical twin?”
7. Pair groups and have them exchange their custom Phase B designs. Ask them to test each other’s patterns using a transparent ruler.
8. Facilitate by saying:
“Trade papers with a neighboring group. Do not simply take their word for it. Measure their pattern using your ruler. If they used a 2 cm glide reflection, locate the original motif, skip the flipped one, and find its identical twin. Is it exactly 4 cm away?
9. Work together to justify why the pure translation vector is always exactly twice the glide distance.”
10. Bring the class back together and display examples of frieze patterns used in modern industries.
11. Synthesize the lesson by asking:
“Why does this mathematics matter? Graphic designers, digital animators, and architects do not draw long border patterns by hand. Instead, they create a single motif and program a translation vector into their software to repeat it infinitely.”</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS:
FRIEZE PATTERN ANALYST

Instructions:
1. Use a strip of bond paper or a digital drawing tool. Your draft must include the following elements.
2. Motif: draw and label the motif or smallest repeating unit.
3. Four repetitions: repeat the motif at least four times along one direction to form a Frieze strip.</td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS:
FRIEZE PATTERN ANALYST

1. Initiates the session by displaying authentic textile Frieze patterns (e.g., Inabel weaves, Malong borders, or modern printed sleeves). The teacher points out that behind every beautiful design is a consistent mathematical "recipe."
2. Emphasize that while creativity is encouraged, the design must be accurate, continuous, and structurally consistent.
3. Demonstrate that a "broken" pattern in math results in a "factory defect" in design.</td></tr>
</table>

	4. Translation vector: draw an arrow from one motif to the next matching motif, and label the vector (for example, 4 cm to the right, or (4, 0)). 5. Additional symmetries: check whether your strip has any of vertical reflection axis, horizontal reflection axis, glide reflection, or half-turn rotation; label what applies. 6. Mathematical justification (5 to 6 sentences): a. What is your motif? b. What translation vector generates the repeated strip? c. What additional symmetries are present, if any? d. Why does your strip satisfy the definition of a Frieze pattern? e. How does your Frieze strip connect to your group exhibit theme? Note: before you begin, the teacher will display the Performance Task checklist (motif, at least four repetitions, translation vector, additional symmetries, written justification). You must draw the translation vector; a statement such as "it repeats" is not enough.	4. As learners sketch, the teacher circulates and poses reflective questions to move beyond simple drawing: a. *"Which transformation makes your design look most dynamic?"* b. *"How could you describe your pattern to someone else so they could recreate it exactly?"* 5. Ensure learners effectively use drawing tools and "viewfinders" to isolate their generating units, ensuring the repetition remains identical throughout the strip. 6. Explicitly links the lesson to careers in graphic design and architecture, emphasizing that spatial reasoning is a high-value skill in the global job market. 7. To foster a learner-centered culture, the teacher facilitates a "Design Review" where learners share their work and peers must identify the transformations used, reinforcing vocabulary through social learning.
	C.2. Making Generalization	
	Activity C.2. **WRAPPING UP THE LESSON: 3-2-1 EXIT SLIP** **Instructions:** Complete the 3-2-1 activity. First, write down 3 transformations that you used or observed in Frieze patterns. Next, write 2 real-world examples of Frieze patterns. Finally, write 1 sentence in your own words that defines what a Frieze pattern is. Take your time to think carefully and hand in your exit slip when you are done.	**Activity C.2.** **WRAPPING UP THE LESSON: 3-2-1 EXIT SLIP** To summarize the lesson, facilitate the provided activity.
	C.3. Evaluating Learning	
	Activity C.3. **ASSESSING LEARNING OUTCOMES: FRIEZE PATTERN EXIT TASK** Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Guide learners in identifying the motif, translation vector, and any reflections, rotations, or glide reflections present in each problem. 2. Encourage learners to explain their reasoning by asking questions such as, *"What evidence supports your answer?"* and *"What transformation maps one motif to the next?"* 3. Monitor learners' work and address misconceptions, particularly that repetition alone does not create a Frieze pattern and that a glide reflection involves both a reflection and a translation. 4. Invite learners to present and justify their solutions using appropriate geometric terms and transformations.	

	5. Emphasize that every Frieze pattern must have a fixed translation, while other symmetries may or may not be present depending on the pattern.
	C.4. Additional Activities
	Activity C.4. **EXTENDING AND REINFORCING LEARNING: DIGITAL FRIEZE QUEST** Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Introduce the **Digital Frieze Quest** and explain that learners will apply their understanding of Frieze patterns by identifying examples found in their home, school, or community. 2. Instruct learners to photograph or sketch a Frieze pattern and analyze the transformations present, including translation, reflection, rotation, or glide reflection. 3. Encourage learners to observe how geometric transformations are used in real-world designs such as fences, floor tiles, fabrics, decorative borders, and architectural details. 4. Review learners' submissions and provide feedback on the accuracy of their classification and identification of transformations, addressing any misconceptions as needed. 5. Facilitate a brief sharing session during the next class meeting to allow learners to present their findings and appreciate the application of mathematics in everyday life.

V. ASSESSMENT	**To evaluate learners' success in attaining the intended learning competencies, the assessment tools and strategies provided on the link in the table of contents can be utilized to measure understanding, skills, and application of concepts**
VI. REFLECTION	**To assess and evaluate the effectiveness of the instruction, as well as to identify challenges and plan for improvements in this unit, teachers are encouraged to answer the reflective questions provided in the link indicated in the table of contents**

UNIT 2. PATTERNS IN NATURE AND ARTS

I. LEARNING GOALS	
Content Standard	The learner demonstrates understanding of ... 1. Golden Ratio in nature and art. 2. Fibonacci Sequence in nature and art. *3.* the fractals in nature and art.
Performance Standard	By the end of the quarter, the learners are able to contribute to a class exhibit of artworks inspired by geometric symmetries, tessellations, Frieze patterns, golden ratio, Fibonacci sequence, or fractals. The exhibit should be accompanied with a description that details the mathematics in the artwork.
Learning Competencies	The learners ... 1. define and explain the Golden Ratio, Golden Rectangle, Golden Spiral, and Golden Angles. 2. identify the golden ratio, golden rectangle, golden spiral, and golden angles in nature and art 3. define the Fibonacci sequence and investigate its historical origin. 4. identify examples of Fibonacci sequence in nature and art 5. explain the connection between the Golden Ratio and the Fibonacci sequence 6. define and identify fractals in nature and art.

	7. generate fractals with or without the use of technology.
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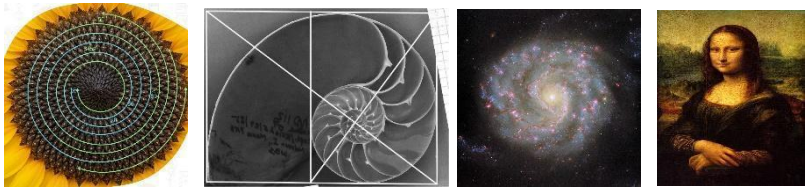
Materials	• Learning Activity Sheet or worksheet • Printed images (natural objects, artworks, spirals, flowers, plants) • Printed activity sheets/worksheets • Manila paper / Chart paper • Marker pens • Geometric tools (rulers, compasses, graphing paper) • Paper cut-outs (Thumbs Up/Down, mature and baby rabbits) • Metacards • Drawing and coloring materials
EdTech	• Geogebra • Khan Academy Platform. "Mathematics resources" & "Golden ratio in nature and art." https://www.khanacademy.org • PowerPoint presentation media • Canva • Projector • Laptop / Computer • TV set • Smartphones / Mobile devices
Use of Generative AI	In preparing this Learning Exemplar (LE), the author(s) used generative artificial intelligence (AI) tools to assist in organizing ideas, developing sample activities, and generating formative assessment items. The author(s) carefully reviewed, validated, and finalized all content. The author(s) take full responsibility for the content of this LE.

III. CONTENT	**Lesson 2.1. THE GOLDEN RATIO IN NATURE AND ART**	
IV. OBJECTIVES	**At the end of the lesson, the LEARNERs should be able to:** **1. describe the golden ratio, golden rectangle, golden spiral, and golden angle using appropriate mathematical language and symbols and as observed in nature, architecture, and artworks;** **2. explain the relationships among the golden ratio, golden rectangle, golden spiral, and golden angle in terms of proportion and pattern formation;** **3. present real-life examples of the golden ratio in plants, shells, animals, architecture, and artworks based on their patterns and proportions; and** **4. construct models of the golden rectangle and golden spiral, and represent the golden angle using appropriate tools and measurements.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge***	
	Activity A.1. **LEVELING LEARNER READINESS: SPOT THE PATTERN**	**Activity A.1.** **LEVELING LEARNER READINESS: SPOT THE PATTERN**

	Instructions: Look carefully at the images shown for one minute, noticing the shapes, patterns, and proportions. Then, write three observations in your notebook. Pair up with a seatmate and share your observations, talking about what is similar or different. Together, pick one pattern you think is common in the images. Be ready to share your pattern with the class. Finally, listen to other pairs and help identify repeating ideas such as spirals, balance, and symmetry.  **Processing Questions**: 1. What do you see in the images? Name the shapes, lines, or patterns. 2. Have you seen these shapes or patterns somewhere before, in nature or art? Give examples. 3. What do these patterns remind you of, or what ideas come to mind when you look at them? 4. Why do you think these patterns appear in both nature and human-made art?	1. Present images such as a sunflower, nautilus shell, spiral galaxy, and selected artworks to capture learners' attention and activate prior knowledge. 2. Explain the task clearly and instruct learners to observe carefully, focusing on shapes, patterns, proportions, symmetry, and spirals, and to prepare to describe what they notice. 3. Guide learners by prompting them on what to look for, such as repeating patterns, symmetry, balance, and natural or artistic structures, without directly giving answers. 4. Encourage learners to work independently first, allowing them to discover patterns on their own before sharing ideas with peers. 5. Facilitate peer discussion where learners compare observations, explain their reasoning, and refine their understanding through interaction. 6. Prompt learners to construct their own understanding by connecting observations from nature and art to mathematical ideas. 7. Ask learners to reflect on their findings and clearly articulate what they observed, ensuring they justify their ideas using evidence from the images. 8. Synthesize learners' responses by linking their observations to key mathematical and artistic concepts, highlighting recurring patterns in nature and design. 9. Guide learners to move from specific examples to generalizations, helping them recognize broader concepts such as symmetry, spirals, and pattern formation in the real world.
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. **APPRECIATING LESSON RELEVANCE: GALLERY WALK** **Instructions**: 1. Form groups of four. 2. Each group will get pictures of buildings, artworks, and things in nature.	**Activity A.2.** **APPRECIATING LESSON RELEVANCE: GALLERY WALK** 1. Facilitate a short discussion by introducing the idea that beauty is often connected to proportion, balance, and harmony in visual forms. 2. Organize learners into small groups of four and distribute printed images of buildings, artworks, and natural objects.

	3. Talk with your group about the question: "Why do people find these images beautiful?" 4. Then, write two reasons on sticky notes and put them on the board under "Beauty and Balance." 5. Finally, read the notes on the board and join the class discussion about the ideas.	3. Prompt each group to discuss the question, "Why do people find these images beautiful?" while encouraging all members to share their ideas. 4. Ask each group to write at least two possible reasons on sticky notes based on their discussion. 5. Instruct groups to post their responses on the board under the heading "Beauty and Balance" and guide a brief class reading of the collected ideas. 6. Lead a short discussion comparing responses and highlighting common themes such as symmetry, proportion, and visual harmony. 7. Explain that artists and architects often use mathematical proportions intentionally to create visually pleasing designs. 8. Conclude by introducing the lesson topic: The Golden Ratio, and connect it to the idea that mathematics helps explain beauty in nature, art, and architecture.
B. Instituting New Knowledge	***B.1. Presenting Example***	
	Activity B.1. **EXPLORING KEY CONCEPTS: THE HOOK AND HUMAN MEASUREMENT LABORATORY** **Instructions:** 1. Form pairs within the class. 2. Obtain a ruler or flexible tape measure and a data recording sheet. 3. Measure and record the following body parts in centimeters: 4. Total height and height from navel to floor 5. Length from shoulder to fingertip and length from elbow to fingertip 6. Length of the entire index finger and length from the middle knuckle to the fingertip 7. Divide each larger measurement by the corresponding smaller measurement. 8. Record all computed decimal values in the worksheet. 9. Compare results with a partner and identify any repeating or similar decimal values. 10. Observe and analyze the patterns formed by the computed ratios. 11. Participate in a class discussion about the observed results. 12. Identify whether many of the ratios are close to approximately 1.6.	**Activity B.1.** **EXPLORING KEY CONCEPTS: THE HOOK AND HUMAN MEASUREMENT LABORATORY** 1. Organize the class into pairs. 2. Distribute rulers or flexible tape measures and a data recording sheet to each pair. 3. Instruct each pair to measure the following body parts and record the measurements in centimeters: 4. Total height and height from navel to floor 5. Length from shoulder to fingertip and length from elbow to fingertip 6. Length of the entire index finger and length from the middle knuckle to the fingertip 7. Direct each pair to divide the larger measurement by the smaller measurement for every pair of values. 8. Record the resulting decimal values in the worksheet. 9. Compare results with a partner and identify recurring decimal values. 10. Facilitate a class discussion by asking learners to observe patterns among the computed ratios. 11. Guide learners to recognize that many ratios are approximately equal to 1.6.

	Processing Questions: 1. What patterns are observed in the ratios obtained from the different body measurements? 2. Why do different body parts tend to produce similar ratio values? 3. How do the results of the measurement activity relate to the idea of the Golden Ratio?	12. Facilitate the processing question as you transition to the discussion.
	B.2. Discussing the Concept	
	Activity B.2. **DEEPENING CONCEPTUAL UNDERSTANDING: THE GOLDEN RATIO IN NATURE AND ART** **Instructions**: The following are important key ideas to be learned, for better understand listen attentively and participate actively during discussion. **The Golden Ratio in Nature and Art** The **Golden Ratio**—often represented by the Greek letter Phi (Φ)is a mathematical constant approximately equal to **1.618**. Two quantities are in the Golden Ratio if the ratio of their sum to the larger quantity is equal to the ratio of the larger quantity to the smaller one. Mathematically, it is expressed as: $$\frac{a+b}{a} = \frac{a}{b} = \Phi$$ By setting $\frac{a}{b} = \Phi$, we can substitute it back into the equation: $$1 + \frac{a}{b} = \Phi \Rightarrow 1 + \frac{1}{\Phi} = \Phi$$ Multiplying the entire equation by Φ yields the characteristic quadratic equation: $$\Phi^2 - \Phi - 1 = 0$$ Applying the quadratic formula and selecting the positive real root, we find the exact value of : $$\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618\ldots$$ When this ratio is applied geometrically, it creates the **Golden Rectangle**. **Golden Spiral**, and **Golden Angle.**	**Activity B.2.** **DEEPENING CONCEPTUAL UNDERSTANDING: DISCOVERING THE GOLDEN RATIO IN NATURE AND ART** **Key Points to Emphasize** A Golden Rectangle is a rectangle whose length-to-width ratio is approximately **1.618**. The Golden Ratio is represented by the symbol **Φ (Phi)**. Precise measurement and careful construction are important in geometry. Golden Rectangles are commonly used in design because of their balanced and visually appealing proportions. **Phase 1. The Phi (Φ) and the Geometry of the Golden Ratio** **Goal:** Connect measurement findings to mathematical concepts and geometric representations. 1. Display diagrams of a Golden Rectangle and a Golden Spiral. 2. Present the value 1.618 and introduce it as Phi (Φ), the Golden Ratio. 3. Explain that the Golden Ratio occurs when the ratio of a whole segment to its larger part is equal to the ratio of the larger part to the smaller part. 4. Demonstrate how a square can be removed from a Golden Rectangle while leaving another Golden Rectangle. 5. Illustrate how repeated removal of squares produces a pattern that forms the Golden Spiral. 6. Clearly explain the Golden Angle. 7. Relate the geometric construction to the ratios observed during the measurement activity.

1. The Golden Rectangle

A **Golden Rectangle** is a rectangle whose side lengths are in the golden ratio, meaning the ratio of the width to the height is exactly $1:\emptyset$.

Mathematical Property: If a Golden Rectangle has dimensions , partitioning a perfect square $a\ x\ a$of dimensions $a\ x\ (a\ +\ b)$ from it leaves a remaining sub-rectangle with dimensions $b\ x\ a$. The ratio of the sides of this new, smaller rectangle remains perfectly proportional:

$$\frac{a}{b} = \Phi$$

This creates a nested, infinitely recurring sequence of smaller Golden Rectangles.

2. The Golden Spiral

Connecting the opposite corners of the nested squares inside a Golden Rectangle with a smooth arc generates the famous **Golden Spiral**.

Polar Equation: In polar coordinates (r, θ), the mathematical equation for a Golden Spiral is: $r = ae^{b\theta}$

Where is the distance from the origin, is the angle of rotation in radians, is an arbitrary scaling factor, and the constant growth factor is defined as:$b = \frac{ln\ (\emptyset)}{\frac{\pi}{2}} = \frac{2\ ln(\emptyset)}{\pi} \simeq 0.3063489$

Because of this specific growth factor, the spiral grows wider by a factor of exactly $\emptyset \approx 1.618$ for every quarter-turn (90^0or $\frac{\pi}{2}$ radians) it completes.

3. The Golden Angle (Ψ)

The **Golden Angle** is created by taking the total circumference of a circle (2πradians) and dividing it into a longer arc and a shorter arc in the Golden Ratio $\left(\frac{a}{b} = \emptyset\right)$. The Golden Angle, denoted by Ψ(Psi), is the central angle subtended by the smaller arc b .

Radian Derivation: $\Psi = 2\pi\left(1 - \frac{1}{\emptyset}\right) = 2\pi\ (2\ -\ \emptyset)$ radians

Degree Conversion: : $\Psi = 360^0\ x\ (2\ -\ \emptyset) \approx 137.50^0$

Golden Ratio Manifests Across Nature, Art, and Design.

1. The Golden Ratio in Nature

8. Briefly discuss the connection between the Golden Ratio and the Fibonacci sequence.

Phase 2. Interactive Exploration Challenge

Goal: Investigate applications of the Golden Ratio in nature, art, and design.

Station 1. Botany Station – Nature's Efficiency

a. Examine images of a sunflower, pinecone, and succulent.
b. Count the clockwise spirals and counterclockwise spirals visible in each image.
c. Record the spiral counts in the worksheet.
d. Compare the numbers obtained with terms in the Fibonacci sequence.
e. Discuss how these patterns contribute to efficient growth and packing.

Station 2: Masterpiece Station – Artistic Intent

a. Examine images of artworks or architectural structures such as the Mona Lisa or the Parthenon.
b. Place a transparent Golden Rectangle or Golden Spiral overlay on the image.
c. Adjust the overlay to align with the composition.
d. Identify where the center of the spiral or key divisions of the rectangle fall.
e. Discuss how artists and architects may use these proportions to emphasize focal points.

Station 3: Marketing Station – Modern Design

a. Examine selected company logos.
b. Analyze the circles, curves, and proportions used in the designs.
c. Measure or estimate relationships among the geometric elements.
d. Identify possible applications of Golden Ratio principles in logo construction.
e. Discuss how balanced proportions may influence visual appeal and recognition.

Phase 3. Consolidation and Debunking Misconceptions

Goal: Summarize learning and distinguish scientific evidence from common myths.

1. Bring the class together for a synthesis discussion.

	In nature, the Golden Ratio is less about a "mystical blueprint" and more about **efficiency**. It is closely tied to the **Fibonacci Sequence** (1, 1, 2, 3, 5, 8, 13...), where dividing any number by its predecessor gets closer and closer to 1.618 as the sequence progresses. Nature utilizes this mathematical pattern to maximize space and energy consumption: • **Phyllotaxis (Leaf and Seed Arrangement):** Look closely at a sunflower head or a pinecone. The seeds grow in clockwise and counterclockwise spirals. The number of spirals almost always equals two consecutive Fibonacci numbers (e.g., 34 and 55 spirals). This exact spacing, dictated by , ensures that every seed gets maximum sunlight and room to grow without crowding out its neighbor. • **Shells and Marine Life:** The chambered nautilus shell grows wider at a constant geometric rate, closely mimicking the logarithmic expansion of the Golden Spiral. This allows the organism to grow larger without changing its fundamental shape or losing structural integrity. • **Hurricanes and Galaxies:** On a grand scale, the spiral arms of a spinning hurricane and the vast swirling arms of spiral galaxies (like our Milky Way) naturally arrange themselves along a logarithmic spiral close to , matching the path of least resistance for moving matter and energy. **2. The Golden Ratio in Art and Architecture** Artists and architects throughout history have gravitated toward the Golden Ratio because the human eye processes images composed around it remarkably fast, subconsciously associating it with balance and beauty. **Classical Architecture** **The Parthenon (Ancient Greece):** While historians debate whether the Greeks used the ratio explicitly, a Golden Rectangle fits almost perfectly around the facade of the Parthenon. The spacing of its columns and the height of its structural elements heavily mirror these proportions.	2. Review the Golden Ratio patterns observed in human measurements, nature, art, and design. 3. Present the misconception that all beautiful objects follow the Golden Ratio. 4. Explain that many natural forms only approximate the Golden Ratio and may follow different mathematical patterns. 5. Discuss how efficient growth, organization, and space utilization often explain the appearance of these patterns in nature. 6. Emphasize that mathematical models describe patterns in nature but do not apply perfectly in all situations.

Renaissance Masterpieces

Leonardo da Vinci: Da Vinci was fascinated by mathematics and illustrated a book titled *De Divina Proportione* ("On the Divine Proportion"). He intentionally incorporated the ratio into his masterpieces:

- ***The Mona Lisa*:** If you draw a Golden Rectangle over her face and a Golden Spiral starting from her wrist, it contours perfectly around her eye and face, guiding the viewer's gaze.
- ***The Last Supper*:** The architectural layout of the room, the alignment of the windows, and the positioning of Jesus and the disciples are mapped out using these geometric constraints.
- Modern Art and Design
- **Salvador Dalí:** In his painting *The Sacrament of the Last Supper*, Dalí explicitly painted the canvas to fit the exact dimensions of a Golden Rectangle. He also chose a massive, translucent regular dodecahedron (a 3D shape built from pentagons, which are deeply tied to the Golden Ratio) to float above the table.

Modern Branding: You see it every day in graphic design. Corporate giants like **Apple**, **Google** (in its G logo), and **Twitter/X** use intersecting circles derived from the Golden Ratio to construct their iconic logos, ensuring visual harmony and instantly recognizable symmetry.

Summary Table: Nature vs. Art

Domain	How Manifests	Primary Function
Nature (Botany)	Sunflower seeds, pinecones, leaf spacing	**Efficiency:** Maximizes space, sunlight, and structural density.
Nature (Physics)	Hurricanes, spiral galaxies	**Dynamics:** Follows the path of least resistance for moving energy.
Classical Art	Paintings (*Mona Lisa*), Architecture (Parthenon)	**Aesthetics:** Creates an intuitive sense of balance and focal harmony.
Modern Design	Tech logos, web layouts, product design	**Clarity:** Allows the human brain to scan and recognize shapes quickly.

<table>
<tr><td></td><td colspan="2">The Golden Ratio serves as a striking bridge between abstract mathematics and the physical world—proving that what is functional in nature is often what we find most beautiful in art.</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3.
PRACTICING LEARNED SKILLS: CONSTRUCTING THE GOLDEN RECTANGLE

Before facilitating this activity, ensure that all learners have a copy of the LAS.
1. Ensure that each learner has a ruler, compass, pencil, and graph paper.
2. Prepare a sample Golden Rectangle construction for demonstration.
3. Review the steps for using a compass accurately and safely.
4. Instruct learners to draw a square measuring 6 cm × 6 cm.
5. Monitor the class and check that all squares are accurately constructed before proceeding.
6. Guide learners in locating and marking the midpoint of one side of the square.
7. Demonstrate how to adjust the compass from the midpoint to the opposite corner of the square.
8. Show how to draw an arc extending beyond the side of the square to determine the length of the Golden Rectangle.
9. Circulate around the classroom to assist learners experiencing difficulty with compass use or geometric construction.
10. Prompt learners to complete the rectangle by drawing the remaining sides.
11. Encourage learners to measure the length and width carefully and record the measurements.
12. Ask learners to compute the ratio of the length to the width.
13. Facilitate a discussion by asking the processing questions.
14. Emphasize that minor variations are expected due to measurement and drawing inaccuracies.
15. Reinforce the idea that mathematical constructions can be used to create aesthetically pleasing and proportionate figures.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
1. Form a group with two classmates.
2. Plan a poster, logo, or simple artwork that incorporates the Golden Ratio.
3. Create the design using a Golden Rectangle, Golden Spiral, or both as a guide for the arrangement of elements.
4. Ensure that the design demonstrates balance, proportion, and visual appeal.
5. Label the parts of the design where the Golden Ratio, Golden Rectangle, or Golden Spiral is applied.
6. Prepare a brief explanation describing how the Golden Ratio influenced the design.
7. Present the completed work to the class.</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

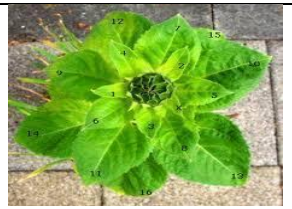
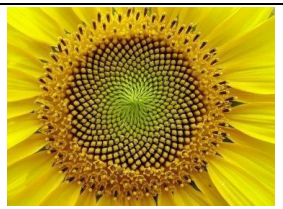
Before the Activity
1. Organize learners into groups of three.
2. Prepare art materials such as bond paper, cartolina, rulers, pencils, markers, colored pencils, and sample templates of the Golden Rectangle and Golden Spiral.
3. Display examples of posters, logos, artworks, and designs that incorporate the Golden Ratio.
4. Review the characteristics of the Golden Rectangle, Golden Spiral, and Golden Ratio.

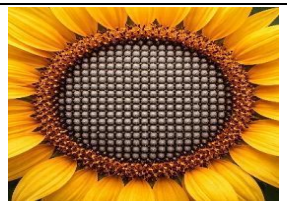
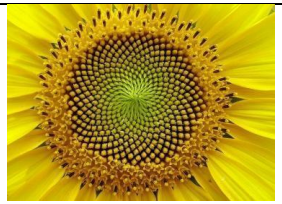
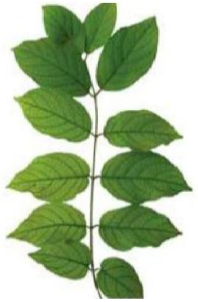
During the Activity
1. Explain the task and expected output.</td></tr>
</table>

	8. Explain the design choices and identify the features that demonstrate the Golden Ratio. **Processing Questions**: 1. How mathematics can be applied in creative fields. 2. Which part of the artwork serves as the focal point? 3. Discuss the role of proportion and balance in effective visual design.	2. Encourage each group to discuss possible design ideas before starting the artwork. 3. Remind learners to intentionally incorporate the Golden Rectangle, Golden Spiral, or both into the composition. 4. Guide groups in planning the placement of images, text, and other design elements using Golden Ratio principles. 5. Circulate around the classroom to monitor progress and provide assistance when needed. 6. Ask guiding questions. 7. Encourage collaboration by ensuring that all group members contribute to the planning, construction, and presentation of the design. 8. Remind groups to label the portions of the design where the Golden Ratio is evident. ~~**Presentation and Processing**~~ 1. Invite each group to present the completed design to the class. 2. Ask groups to explain: 3. The type of design created. 4. How the Golden Ratio was incorporated. 5. The purpose of the design. 6. The features that demonstrate balance and proportion. 7. Encourage classmates to observe and identify the Golden Ratio elements in each presentation. 8. Facilitate a discussion on the similarities and differences among the designs.
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Instructions:** 1. Reflect on the concepts and activities completed during the lesson. 2. Answer the following questions in complete sentences: a. What is the Golden Ratio? b. What makes the Golden Ratio significant in mathematics, nature, art, or design? 3. Identify two examples in nature, art, architecture, or design where the Golden Ratio can be observed.	**Activity C.2. WRAPPING UP THE LESSON** 1. Instruct learners to answer the reflection questions independently. 2. Allow sufficient time for learners to organize and write their responses. 3. Monitor progress and provide clarification when necessary. 4. Encourage learners to support their answers with examples discussed during the lesson. 5. After individual work, direct learners to pair with a classmate and share responses.

	4. Compare responses with a partner. 5. Discuss similarities and differences between the answers. 6. Revise responses, if necessary, based on insights gained from the discussion. 7. Participate in the class sharing and contribute key ideas learned about the Golden Ratio and its applications.	6. Encourage active listening and respectful exchange of ideas during partner discussions. 7. Ask learners to compare answers and identify common ideas and unique insights. 8. Invite learners to revise or expand responses based on the discussion.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Facilitate Activity C.3 using the Learning Activity Sheet. 2. Remind learners to read each question and task carefully before answering. 3. Encourage the application of concepts and skills learned about the Golden Ratio, Golden Rectangle, and Golden Spiral. 4. Monitor learner progress and provide clarification on instructions when necessary without revealing answers. 5. Observe learner responses to identify areas of strength and misconceptions. 6. Gather and review completed Learning Activity Sheets. 7. Provide timely feedback on the accuracy of answers, quality of explanations, and application of mathematical concepts. 8. Discuss common errors and exemplary responses to reinforce understanding. 9. Address misconceptions and clarify concepts that require further development. 10. Use the results to determine whether additional enrichment, remediation, or practice activities are needed.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. The tiered remediation and enhancement activities are designed to address the diverse learning needs, abilities, and pace of learners in the classroom. These activities are anchored on the principles of **differentiated instruction**, ensuring that all LEARNERs are given equitable opportunities to achieve the lesson objectives. For **remediation**, learners who demonstrate difficulty in understanding the concept are provided with simplified, guided, and scaffolded tasks. These include step-by-step procedures, visual aids, and concrete examples to strengthen foundational knowledge and skills. The goal is to build confidence and ensure mastery of essential competencies. For **enhancement**, learners who demonstrate proficiency are given more complex, open-ended, and creative tasks. These activities encourage higher-order thinking skills such as analysis, synthesis, and application. Learners are challenged to extend their understanding through independent or collaborative work, often integrating creativity and real-world connections. The use of **tiered tasks (beginner, intermediate, and advanced levels)** allows the teacher to adjust the level of difficulty while maintaining the same learning objective. This ensures that all learners remain engaged, appropriately challenged, and supported throughout the lesson.	

III. CONTENT	Lesson 2.2. FIBONACCI SEQUENCE	
IV. OBJECTIVES	**At the end of the lesson, learners should be able to:** **1. describe the Fibonacci sequence and describe the rule for generating its terms.** **2. investigate and explain the historical origin of the Fibonacci sequence.** **3. generate terms of the Fibonacci sequence.** **4. identify occurrences of Fibonacci numbers in plants, flowers, shells, and artworks.** **5. illustrate or present examples of Fibonacci sequences found in nature and art through drawings, diagrams, or digital media.** **6. explain how the ratio of consecutive Fibonacci numbers approximates the golden ratio.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILIATATION GUIDE)**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS **Instructions**: 1. Form groups of five learners. 2. As a team, analyze the sequence of numbers. Then identify the rule that generates the next term. a. 2, 5, 8, 11, 14, 17 b. 4, 12, 36, 108, 324 c. 240, 120, 60, 30, 15 d. 1, -1, 1, -1, 1, -1 e. 1, 4, 9, 16, 25, 36 3. Observe the pictures provided below and answer the questions that follow. Image 1	**Activity A.. LEVELING LEARNER READINESS** Divide the class into groups of five. Provide each group with a copy of the learning activity. Allow them up to 10 minutes to discuss and accomplish the task collaboratively. During the checking of answers, let LEARNERs share their strategies in arriving at their responses. Challenge them to express the rule in mathematical form. Distribute a set of images to each group and provide time for them to observe the pictures carefully. In 5 minutes, let them discuss their observations within their group and then share their insights with the class. Facilitate the discussion of their responses to the processing questions and connect their ideas to the current lesson topic. Proceed to processing questions. **Processing questions:** 1. What type of sequence is presented in the activity? 2. What rule governs each sequence? How did you determine it? 3. Which sequence was easiest? hardest? Why? 4. How are sequences applied in real-life situations? 5. Do you notice any patterns or repetitions? State your observation. 6. Can you predict the next element in the pattern of Image 7. Where else might this kind of pattern appear in real life?

	 Image 2 Image 3  Image 4	8. How do the patterns shown in the flowers, leaves, pinecone, and sunflower demonstrate mathematical relationships in nature? 9. How can you apply this pattern to design something efficient or aesthetically pleasing in real life?
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. APPRECIATING LESSON RELEVANCE **Instructions:** Perform the tasks provided**.** **Part I.** Read the short article below and answer the questions that follow. **Article:** Nature often follows mathematical patterns, and one of the most remarkable is the Fibonacci sequence. This sequence is not merely a mathematical curiosity; it appears in the growth and arrangement of many plants. In a sunflower, seeds are arranged in spirals radiating from the center. The number of spirals in each direction often corresponds to consecutive Fibonacci numbers. Similarly, the scales of a pineapple grow in spiral rows that follow the Fibonacci sequence. In leaf arrangement, known as phyllotaxis, plants position their leaves at angles approximating the golden angle (about 137.5°), producing spiral patterns associated with Fibonacci numbers. **Questions:** Why is the Fibonacci sequence important in the arrangement of: a. seeds in sunflowers? b. scales in pineapples? c. leaves in plants? **Part II**. Study the two sets of images. Then answer the specified question. **Set 1**	**Activity A.2. APPRECIATING LESSON RELEVANCE** This activity will get LEARNERs to think for themselves, discuss with a partner, and share with the whole class to help them understand the Fibonacci sequence in nature better. This process helps LEARNERs understand how math relates to real-life plant patterns and gets them ready for the lesson's objectives. **Part I. Think-Pair-Share** 1. Distribute a copy of the article to each of the LEARNERs. Give them 2 minutes to choose a question and write their answer on a sheet of paper. 2. Instruct LEARNERs to discuss their answers with a partner. 3. Pairs share their answers to the whole class. 4. Wrap up the discussion by highlighting the importance of the Fibonacci sequence in plants, particularly in relation to optimal packing efficiency and patterns of growth and development. Afterward, present the lesson objectives. **Part II.** Post each set of images on the board together with the focus question. Allow LEARNERs to paste a pre-prepared Thumbs Up or Thumbs Down clipart below each image to indicate their response. Afterward, call 2–3 LEARNERs to share their answers to the focus question. Wrap up the discussion by highlighting the importance of the Fibonacci sequence in plants, particularly in terms of optimal packing efficiency and patterns of growth and development. Then, present the lesson objectives.

<table>
<tr><td></td><td> 
Which sunflower has the better packing efficiency of seeds? Why?
Set 2
 
Which leaf arrangement has better access to sunlight? Why?</td><td></td></tr>
<tr><td rowspan="2">B. Instituting New Knowledge</td><td colspan="2">B.1. Presenting Examples</td></tr>
<tr><td>Activity B.1. EXPLORING KEY CONCEPTS

Instructions:
In this activity, you will act as a mathematical investigator tasked with uncovering a pattern in the growth of a rabbit population, similar to the problem introduced by Leonardo Fibonacci in his book Liber Abaci.

Scenario:
A man starts with one pair of rabbits in an enclosed area. He wants to determine how many pairs will be produced in one year, assuming that each pair produces another pair every month starting from the second month after birth.

Tasks:
1. Investigate how the rabbit population grows over time and identify the pattern in the number of rabbit pairs.
2. Create a table showing the number of rabbit pairs for each month. Include the following columns:
• Month Number</td><td>Activity B.1. EXPLORING KEY CONCEPTS

Strategy 1. Mathematical Investigation
1. Divide the class into groups of five LEARNERs.
2. Provide each group with the scenario, tasks, and guiding questions.
3. Explain the activity objectives, expected output, and allotted time for completion which is 40 minutes.
4. Ask each group to present their findings before the class. They should describe the pattern they observed, the rule they formulated, and explain how the rabbit population grows over time.
5. Wrap up the discussion by explaining that the total number of rabbit pairs per month follows a mathematical pattern known as the Fibonacci sequence.

Strategy 2. Illustrating Rabbit Pairs Population
1. Prepare cut-outs of rabbit pairs (15 medium size and 10 small size) and 6-7 pieces of Manila papers.</td></tr>
</table>

	• Number of Mature Rabbit Pairs • Number of New Rabbit Pairs Born • Total Number of Rabbit Pairs 3. Analyze the data in your table and observe any patterns in the sequence of numbers. **Processing Questions**: 1. How does the total number of rabbit pairs in the current month relate to the previous months? 2. What pattern do you observe in the way the totals increase? State your observation clearly.	 Mature rabbits Baby Rabbits 2. Divide the class into groups of 5 LEARNERs. Provide them the rabbit cut-outs and the Manila paper. 3. Ensure that every group has marking pen and glue. 4. Instruct the learners that they are going to create the family tree diagram of rabbits given the following scenario: 5. A certain man had one pair of rabbits together in a certain enclosed place, and one wishes to know how many are created from the pair in one year when it is the nature of them in a single month to bear another pair, and in the second month those born to bear also. 6. Provide the standards of the family tree diagram 7. Each level of the family tree diagram corresponds to a month number that is written in the left side of the diagram. 8. On its right, the total number of rabbit pairs must be indicated. 9. Let the LEARNERs analyze the data recorded and write their answers the process questions below the diagram. 10. Each group will post their answers on the classroom walls. learners will then do Gallery Walk to check their other group's answers. 11. Wrap up the discussion by telling the LEARNERs that the total number of rabbit pairs per month form the Fibonacci sequence.
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Instructions**: Study the key concepts, formula, and examples to ponder. For better	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** **Key Points to Emphasize** 1. The Fibonacci sequence follows a specific recursive pattern in which each term is the sum of the two preceding terms.

understanding of the Fibonacci sequence listen attentively and participate actively in the discussion.

The Fibonacci Sequence

The **Fibonacci sequence** is a sequence of numbers in which the first two terms are 0 and 1. Each succeeding term is obtained by adding the two preceding terms.

In math notation, if F_n is a term in the sequence such that n > 2, F_n is defined as follows:

$$F_n = F_{n-1} + F_{n-2}$$

The formula provides the pattern of the Fibonacci sequence, that is, a term (n > 2) is the sum of the two terms that immediately precede it. The sequence begins with 0 and 1, and the rest follows.

The first 8 terms of the Fibonacci sequence is
0, 1, 1, 2, 3, 5, 8, 13

$$F_1 = 0$$
$$F_2 = 1$$
$$F_3 = F_2 + F_1 = 1 + 0 = 1$$
$$F_4 = F_3 + F_2 = 1 + 1 = 2$$
$$F_5 = F_4 + F_3 = 2 + 1 = 3$$
$$F_6 = F_5 + F_4 = 3 + 2 = 5$$
$$F_7 = F_6 + F_5 = 5 + 3 = 8$$
$$F_8 = F_7 + F_6 = 8 + 5 = 13$$

Example 1.
Find the 10th and 13th term of the Fibonacci sequence.

Solution: Create a table of values for n and F_n and use the pattern in finding the value of F_n.

n	1	2	3	4	5	6	7	8	9	10	11	12	13
F_n	0	1	1	2	3	5	8	13	21	34	55	89	144

Therefore, $F_{10} = 34$ and $F_{13} = 144$.

Example 2.

2. Fibonacci numbers frequently appear in natural growth patterns.
3. The Fibonacci sequence is closely related to the Golden Ratio.
4. Mathematical concepts can be represented through problem-solving, historical analysis, observation, artistic creation, writing, and performance.
5. Mathematics is interconnected with many aspects of everyday life and other fields of study.

To facilitate this part of the lesson, use the following strategy:
The Fibonacci Sequence- Explicit Teaching
Prepare all the necessary materials such as slide presentation or other instructional materials. Then discuss the key concepts of the Fibonacci sequence and provide the 3 illustrative examples. Provide additional examples if necessary. Then let the LEARNERs answer the processing questions.

The Fibonacci Sequence- Guided Discovery
1. Present the first eight terms of the Fibonacci sequence to the learners.
2. In 2 minutes, ask them to examine the numbers, describe the sequence, and identify the pattern.
3. Employ the Think-Pair-Share technique to facilitate discussion.
4. Address misconceptions and formalize the key concepts.

The Fibonacci Sequence - I Do, We Do, You Do Technique
I DO: The teacher discusses the key concepts and provides the first example.
WE DO: The teacher guides the learners in solving the second example.
YOU DO: Let the LEARNERs solve the third example and answer the processing questions. Given them at most 5 minutes.

Brief History of Fibonacci Sequence - Concept Mapping
This will be done individually in an A4 bond paper. With Fibonacci Sequence at the center of the map, let the

	Which of the following set of numbers is part of the Fibonacci sequence? A. 233, 377, 610 B. 144, 237, 381, 618 C. 987, 1597, 2584 Solution: Set A = 233, 377, 610 belongs to the Fibonacci sequence since 610 = 377 + 233 and the earlier terms follow the starting pattern, that is, 377 – 233 = 144 21 – 13 = 8 233 – 144 = 89 13 – 8 = 5 144 – 89 = 55 8 – 5 = 3 89 – 55 = 34 5 – 3 = 2 55 – 34 = 21 3 – 2 = 1 34 – 21 = 13 2 – 1 = 1 1 – 1 = 0 Set B = 144, 237, 381, 618 does NOT belong to the Fibonacci sequence though the term was the sum of the two preceding terms, he it failed to follow the starting pattern of the Fibonacci sequence. Set C = 987, 1597, 258 is part of the Fibonacci sequence. If you subtract the first two terms, you will get a certain number. If you continue the process, you will obtain the starting pattern of the Fibonacci sequence, just like in Set A. **Processing Questions**: 1. How did you determine the 10th and 13th terms of the Fibonacci sequence? What pattern did you follow? 2. What rule or relationship helped you identify which set of numbers belongs to the Fibonacci sequence? 3. Why is recognizing patterns important in solving problems involving sequences like the Fibonacci sequence? **Example 3**. Consider the word problem below. Rica, a young entrepreneur in La Trinidad, Benguet is starting a small strawberry nursery business. To model inventory growth, she decided to follow the Fibonacci sequence in propagating strawberry	LEARNERs write their answers to the Process questions at the branches of the map. Allotted time is at most 15 minutes. **Brief History of Fibonacci Sequence - Math Talks** A representative of each group of LEARNERs will act out as Leonardo of Pisa and explain to the whole class how he discovered the Fibonacci sequence. Give the LEARNERs at most 10 minutes to prepare. **Fibonacci Sequence in Nature and Art- Video Notes** Provide the LEARNERs metacards where they will write their answers to the process questions. After 10 minutes, call volunteers to share their answers to the whole class. **Fibonacci Sequence in Nature and Art- Math Concept Wall** Using Manila papers, create 3 concept walls for the three process questions where LEARNERs will write their answers. After 10 minutes, check LEARNERs' answers, and establish the essential concepts. **Connection of Fibonacci Sequence to Golden Ratio Think-Pair-Share** This activity may be done in pairs or in groups.learners will complete the five steps individually, then share their answers with a partner or their group. After 10 minutes, call 2–3 volunteers to present their discoveries to the class. Provide feedback and synthesize the key concept. **Connection of Fibonacci Sequence to Golden Ratio - Amazing Math Relay** 1. Create 5-7 work stations in front of the classroom. Add more depending on how many groups the learners have. The 5 steps of the math discovery activity are placed in the work station. Assign advanced learners as station facilitators. 2. At a certain distance, learners line up opposite their work station. The first learner in the line will perform Step 1, second learner will perform Step 2, and so on.

	plants. If she starts with 5 runners in the first month and 8 runners in the second month, how many runners will she plant in the 6th month? **Questions**: 1. Create a table showing the number of runners Rica have planted over 6 months. 2. How many runners will Rica plant in the 6th month? 3. How can recognizing the pattern help Rica in managing her planting and future supply? **Brief History of Fibonacci Sequence** Read or watch the short story about the man behind Fibonacci sequence and how he discovered these amazing numbers. https://www.youtube.com/shorts/YHrTfND8lac The Fibonacci sequence, commonly attributed to medieval Italian mathematician Leonardo Fibonacci (c. 1170-1250), has a rich history spanning multiple civilizations and millennia. As a young man attending to his family's trade in Africa, Fibonacci, also known as Leonardo of Pisa, studied mathematics from Islamic teachers. From them, he learned of the Hindu-Arabic numerals and their computing algorithms as used in business transactions. Impressed by this new knowledge, he returned to Pisa where he disseminated it by writing *Liber abaci* (Book of Calculation) in 1202. Fibonacci initially discovered this sequence while studying rabbit population growth under ideal conditions. The problem posed was, if we start with a pair of rabbits, how many pairs would there be after a year if each pair produces a new pair every month and new pairs become productive after two months? This seemingly simple question led to one of mathematics' most influential sequences. The sequence of numbers in the Fibonacci sequence describes the population increase in rabbits for *n* months. Fibonacci discovered that the number of pairs of rabbits. for any month is the sum of the numbers of pairs from the two preceding months. **Processing Questions**:	3. Each learner-player is given at most 3 minutes to perform the task assigned to him/her. If a player failed to accomplish the task within the given time, he returns to the base line and the next player will continue the task. 4. Conclude the activity by checking learners' answers and formalizing the key concepts developed from the activity.

	1. From what country is Leonardo? 2. How did being a merchant's son help him in his mathematical endeavors? 3. What specific conditions regarding rabbit reproduction and productivity did Leonardo of Pisa use to discover the sequence? 4. Explain how did he discover the Fibonacci sequence? **Fibonacci Sequence in Nature and Art** You may read the articles or watch the short video clips about Fibonacci sequence in nature and art. https://www.youtube.com/shorts/nWC7tKIArRA https://www.youtube.com/shorts/7vyN36uFIuM https://www.youtube.com/shorts/3e_x-DLRGDA Beyond mathematics, Fibonacci sequence appears widely in nature and art, revealing how patterns and order exist in the world around us. In nature, it can be seen in the arrangement of seeds in a sunflower allowing the seeds to pack tightly and evenly, the spiral patterns of pinecones, and the structure of seashells. It is also especially evident in phyllotaxis, or the arrangement of leaves around a stem. Leaves grow at specific angles that follow the Fibonacci sequence, helping each leaf receive enough sunlight without blocking others. These natural formations often follow Fibonacci numbers because they allow for efficient growth, maximizing space and resources. The influence of the Fibonacci sequence also extends to art and design. Artists and architects use this pattern to create works that are visually pleasing and well-proportioned, often connected to the golden ratio. From spiral artworks to building designs, the Fibonacci sequence helps achieve harmony and balance. *Then answer the process questions below.* 1. What examples of the Fibonacci sequence did you notice in nature or art from the video, and how were the numbers or patterns shown? 2. How does the Fibonacci sequence help explain patterns such as spirals, arrangements, or proportions in natural objects and artistic designs?	

	3. Why do you think the Fibonacci sequence appears frequently in nature and art, and how might understanding this pattern be useful in real-life fields such as design, architecture, or biology? **Connection of Fibonacci Sequence to Golden Ratio** A mathematical pattern can be observed in both nature and human-made designs. This pattern connects the Fibonacci sequence with the Golden Ratio. You will explore how Fibonacci numbers relate to the Golden Ratio and understand why this relationship appears in spirals, leaf arrangements, shells, and various forms of art and design. Steps: 1. List the first 20 non-zero terms of the Fibonacci sequence. 2. Select consecutive pairs of terms from the sequence. 3. Divide each larger term by its preceding term. 4. Record the quotients from the 1st to the 20th term. 5. Analyze the quotients. Do they approach a specific number? State your observation. **Processing Questions:** 1. What pattern do you notice in the results when you divide consecutive Fibonacci numbers? 2. What number do the quotients seem to get closer to as the sequence continues? 3. How can this pattern or relationship be useful in real life, such as in art or design? The Fibonacci sequence is connected to the Golden ratio. When you divide two consecutive Fibonacci numbers, the quotients approach the Golden Ratio (Φ) that is 1.618. Considering the first 15 non-zero terms (*n*) of the Fibonacci sequence, we have the following quotients of the larger term and the smaller term, rounded to 3 decimal places: ● 1 ÷ 1 = 1 ● 2 ÷ 1 = 2 ● 3 ÷ 2 = 1.5 ● 5 ÷ 3 = 1.667 ● 8 ÷ 5 = 1.600 ● 55 ÷ 34 = 1.618 ● 89 ÷ 55 = 1.618 ● 144 ÷ 89 = 1.618 ● 233 ÷ 144 = 1.618 ● 377 ÷ 233 = 1.618	

- $13 \div 8 = 1.625$
- $21 \div 13 = 1.615$
- $34 \div 21 = 1.619$
- $610 \div 377 = 1.618$
- $987 \div 610 = 1.618$

From the quotients obtained, we can observe that the ratio of two consecutive Fibonacci numbers becomes progressively closer to the Golden Ratio (Φ) as n increases.

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.

1. Prepare task cards labeled **Task 1 to Task 6**.
2. Ensure the availability of materials such as paper, markers, art supplies, reference materials, and digital devices (if applicable).
3. Arrange the classroom to allow group collaboration and presentations.
4. Review the mechanics, expected outputs, and assessment criteria with the class.
5. Organize the class into groups of five.
6. Instruct each group to select a representative.
7. Allow each representative to randomly pick a task card.
8. Provide sufficient time for groups to discuss, plan, and complete the assigned task.
9. Circulate around the classroom to monitor progress, answer questions, and provide guidance.
10. Encourage equitable participation among group members.
11. Observe how learners apply concepts related to the Fibonacci sequence and the Golden Ratio.

Task-Specific Facilitation

Task 1. Fibonacci Challenge

1. Encourage learners to review the rule of the Fibonacci sequence before solving the problem.
2. Prompt groups to verify each term by adding the two preceding terms.
3. Ask learners to justify the correctness of the sequence using mathematical reasoning.
4. During reporting, encourage groups to explain the verification process rather than simply providing the answer.

Task 2. His Story of Fibonacci

1. Guide learners in organizing information about Leonardo of Pisa and *Liber Abaci*.
2. Encourage the use of simple language and engaging dialogue.
3. Remind learners to include the rabbit population problem and its connection to the Fibonacci sequence.
4. During the presentation, observe the accuracy and clarity of the historical explanation.

Task 3. Nature Pattern Hunt

1. Provide images or allow observation of available natural objects.
2. Encourage careful examination of spirals, petals, leaves, and seed arrangements.
3. Ask learners to identify and record Fibonacci numbers observed in the patterns.

<table>
<tr><td></td><td colspan="2">4. Facilitate discussion on why these patterns may support efficient growth and space utilization.

Task 4. Fibonacci Art
1. Encourage learners to incorporate Fibonacci numbers, Golden Rectangles, or Golden Spirals into the design.
2. Remind groups to label mathematical elements clearly.
3. Ask learners to explain how mathematical proportions contribute to the visual appeal of the artwork.
4. During presentations, focus on both creativity and mathematical accuracy.

Task 5. Fibonacci Journal
1. Explain that each line should follow the word counts represented by Fibonacci numbers.
2. Encourage learners to reflect on concepts learned about the Fibonacci sequence.
3. Monitor the structure of the journal entries to ensure adherence to the sequence.
4. During sharing, ask learners how the activity influenced their understanding of mathematical patterns.

Task 6. Jingle Presentation
1. Encourage learners to summarize essential concepts of the Fibonacci sequence through lyrics and movement.
2. Remind groups that mathematical content should remain accurate and meaningful.
3. Allow rehearsal time before presentations.
4. Observe how effectively the presentation communicates key concepts.</td></tr>
<tr><td rowspan="2">C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
Part I. Using your smartphones or mobile devices, take a photo of a plant that exhibit Fibonacci patterns. Place the image on an A4 bond paper and include the following details:
1. Name of plant
2. Scientific name
3. Local name
4. Date captured
5. Location where it was captured
6. GPS coordinates
7. Explanation of the Fibonacci sequence exhibited:

Part 2: In another A4 bond paper, draw an artwork that incorporates the Fibonacci sequence. Below the artwork, explain how Fibonacci sequence and Golden ratio were used in creating your artwork.</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

This activity may be done inside the school or may be a take-home activity. If it is done inside the school, ensure that the learners will surrender their smartphones to you and will only be used only during the class period. After the activity, collect again the smartphones for safekeeping.

The first two parts of the activity will run for at most 60 minutes. After the LEARNERs have accomplished their task, instruct them to post their works on the classroom walls for the class exhibit. Give them 10-15 minutes for the Gallery Walk and have them answer the Processing questions afterwards.

Processing questions:
1. How did you find the Fibonacci sequence in the plants you captured?
2. Why do you think plants follow the Fibonacci sequence?
3. How can you use the Fibonacci sequence to create something useful or beautiful in real life?</td></tr>
</table>

	Part 3: Post your creative outputs on the walls of the classroom for the class exhibit. You are given 10-15 minutes for a Gallery Walk and then answer the following questions:	
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Option 1. Reflection Poster** 1. On an A4 bond paper, create a reflection poster that includes the following: 2. A short paragraph summarizing what you have learned about the Fibonacci sequence 3. At least two illustrations or images showing Fibonacci patterns in nature or art 4. Labels indicating Fibonacci numbers or the Golden Ratio, where applicable 5. A personal reflection on why understanding these patterns is important or interesting **Option 2. 3-2-1 Approach** Using a whole sheet of paper, write: 3 - essential concepts that you learned 2 - real-life situations where you can apply the learned concepts 1- concept that needs clarification	**Activity C.2. WRAPPING UP THE LESSON** In this activity, learners are encouraged to consolidate their understanding of the Fibonacci sequence by reflecting on its key ideas and connections to real-life situations. Through the suggested strategies, they will organize their learning, identify important concepts, and express how these mathematical patterns relate to the world around them. Furthermore, this will help them deepen their understanding and clarify remaining questions about the topic.
	C.3. Evaluating Learning	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. This part of the lesson will allow learners to apply their understanding of the Fibonacci sequence and the Golden Ratio by analyzing real-world images and determining whether they reflect these mathematical patterns. They will also extend their learning by solving a practical problem that shows how Fibonacci-based designs can be applied in engineering and technology. This activity should be completed by individual learners to assess their understanding of the lesson. Prepare a worksheet for every learner.	
	C.4. Additional Activities	
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS.	

	Remediation: Fibonacci Number Sort In this activity, post on the board at least 30 numbers placed inside a rectangular box. The numbers include 20 terms of the Fibonacci sequence and 10 numbers that are not part of the Fibonacci sequence. After the pair sharing, call volunteers to share their answer to the whole class. Give immediate feedback for any misconception. **Enhancement: Fibonacci Design Project** The Enhancement activity is a scaffold for the Performance Task. It may be accomplished at home.

III. CONTENT	**Lesson 2.3. FRACTALS**	
IV. OBJECTIVES	**At the end of the lesson, the learners are expected to:** **1. describe fractals using mathematical terms, highlighting self-similarity, recursion, and infinite complexity;** **2. identify examples of fractals in natural objects (trees, clouds, mountains, rivers) and artworks (digital art, mosaics, patterns);** **3. analyze how fractal properties appear in both natural and man-made designs;** **4. construct simple fractal patterns manually using geometric shapes, repetition, or recursion;** **5. use technology (e.g., drawing software or fractal generators) to create more complex fractal designs; and** **6. apply creativity and mathematical understanding to produce an original fractal artwork suitable for the class exhibit.**	
V. PROCEDURES	**ACTIVITIES**	**ANNOTATION**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. **ASSESSING LEARNER READINESS:** **PATTERN HUNT IN NATURE** **Instructions**: 1. Carefully observe the images displayed by the teacher, which may include tree branches, lightning, clouds, mountains, and river systems. 2. Write at least two observations about the shapes and patterns you see. 3. Work with a partner and compare your observations. 4. List the patterns you noticed, such as branching structures, repeating shapes, or similar formations. 5. Selected LEARNERs will share their answers with the class. **Processing Questions**: 1. What types of patterns did you observe in the images? Describe them. 2. Which patterns appeared repeatedly across different natural objects?	**Activity A.1.** **ASSESSING LEARNER READINESS:** **PATTERN HUNT IN NATURE** 1. Prepare a slideshow or printed images showing natural fractal-like patterns. 2. Display one image at a time and asks the following guiding questions: • What shapes do you observe? • Do the patterns repeat? • What similarities can you identify? 3. Give learners approximately three minutes to observe and record their answers. 4. Ask learners to discuss their observations with a partner. 5. Facilitate the sharing of responses and highlight patterns such as branching structures and repetition. 6. Explain that many natural patterns follow mathematical rules and may be examples of fractals.

	3. How do these patterns demonstrate self-similarity or fractal-like structures? 4. Why do you think these patterns occur in nature? What might be their purpose or function? 5. Can you think of examples of similar patterns in human-made designs or artworks? 6. How do these observations help you understand the relationship between mathematics and nature?	
	A.2. Establishing the Purpose of the Lesson	
	Activity A. 2. APPRECIATING LESSON RELEVANCE **Instructions**: 1. Explore patterns in your everyday surroundings to connect the lesson to real-life contexts. 2. Observe your classroom, school, or home and identify objects, structures, or designs that display patterns, such as tiles, windows, plants, furniture, or posters. 3. Sketch or record at least three patterns they observe. In pairs or small groups, learners compare their observations and discuss which patterns resemble Fibonacci sequences, spirals, or fractal-like structures. 4. Present one interesting example to the class and explain how it connects to the lesson. 5. The teacher facilitates a short discussion linking these observations to nature, art, architecture, and technology, emphasizing the relevance of mathematics in everyday life.	**Activity A. 2. APPRECIATING LESSON RELEVANCE** 1. Present the guide questions. 2. Give learners two minutes for individual thinking and writing. 3. Have learners discuss their answers with their partners for three minutes. 4. Invite volunteers to share their explanations. 5. Affirm correct responses and link them to the concept of self-similarity. 6. Transition to the lesson by stating: "Today we will study a mathematical concept that explains these repeating patterns, called fractals." **Processing Questions:** 1. What patterns did you notice in your surroundings? Describe them. 2. Which of these patterns resemble Fibonacci sequences, spirals, or fractals? 3. How do these patterns relate to nature, art, or technology? 4. Why is understanding these patterns useful in real life? 5. In what ways do humans intentionally use these patterns in design or engineering?
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Instructions**:	**Activity B.1. EXPLORING KEY CONCEPTS** 1. Ask learners to connect their real-life observations to the mathematical concepts of fractals and Fibonacci sequences.

	1. Review the sketches or notes from the previous "Patterns Around Us" activity. Recall patterns you observed that resemble fractals, spirals, or repeating structures. 2. Using the examples provided by the teacher (trees, river systems, shells, sunflowers, mosaics, and architecture), match your observations with the concepts of self-similarity, recursion, infinite complexity, or Fibonacci sequences. 3. In pairs or small groups, learners discuss which patterns best illustrate each mathematical concept and explain why. 4. Each group selects one example to present to the class, showing either a sketch, photograph, or description, and explains the connection to the mathematical concepts. 5. The teacher facilitates a discussion highlighting the connection between natural patterns, artistic designs, and the mathematics behind them, reinforcing relevance and conceptual understanding	2. Guide them in identifying examples of self-similarity, recursion, infinite complexity, and the Golden Ratio in both natural and human-made patterns. 3. Emphasize the relevance of these concepts in nature, art, and technology. 4. Facilitate the discussion highlighting the connection between natural patterns, artistic designs, and the mathematics behind them, reinforcing relevance and conceptual understanding. 5. Provide feedback, clarifies misconceptions, and encourages learners to articulate their understanding of the connection between observation and mathematical theory. **Processing Questions**: 1. Which of your observed patterns resemble fractals, Fibonacci sequences, or golden ratio spirals? 2. How does the selected example illustrate self-similarity, recursion, or infinite complexity? 3. Can you explain why these patterns appear both in nature and human-made designs? 4. How does understanding these concepts help explain the efficiency or aesthetics of these patterns
	B.2. Discussing the Concept	

	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Instructions**: 1. The class will be divided into groups of 4–5 members. 2. Each group receives a set of picture cards showing natural objects (e.g., trees, clouds, river systems, shells) and artworks or human-made designs (e.g., mosaics, spirals, architectural patterns). 3. Carefully examine each picture and discuss its structure, looking for characteristics of fractals such as self-similarity, recursion, or infinite complexity. 4. Each group classifies the pictures into two categories: a. Fractal Patterns b. Non-Fractal Patterns 5. For each classification, learners write a brief justification, explaining why they consider it fractal or non-fractal, citing specific features observed in the image. 6. Selected groups share their classifications and reasoning with the class, while the teacher facilitates discussion to reinforce correct identification and clarify misconceptions. **Processing Questions**: 1. What features helped you determine whether a pattern is fractal or not? 2. How does the pattern show self-similarity, recursion, or infinite complexity? 3. Were there any pictures that were challenging to classify? Why?	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** After observing and connecting real-life patterns to the concepts of fractals, Fibonacci sequences, and the golden ratio, LEARNERs are now ready to apply their understanding by analyzing and classifying patterns. This activity builds on the examples presented earlier and helps LEARNERs deepen their conceptual understanding through hands-on practice. Learners shall identify and classify patterns based on the characteristics of fractals. Guide the discussion by connecting the activity to previous examples, emphasizing self-similarity, recursion, and infinite complexity in natural and human-made patterns. Provide feedback to clarify misconceptions, reinforce conceptual understanding, and encourage LEARNERs to clearly articulate their reasoning. **Definition of Fractals** A fractal is a never-ending, infinitely complex pattern that is created through repetition of a simple process over and over in a continuous feedback loop. These patterns are self-similar across different scales, meaning that smaller parts resemble the whole structure. Fractals are generated through recursion, where a rule or equation is repeatedly applied to produce increasingly detailed structures. Because of this process, fractals are often described as images of dynamic systems or “pictures of chaos,” where simple rules can generate highly complex forms. **Characteristics of Fractals** Fractals have several key properties that make them unique: • **Self-Similarity**: Smaller parts of a fractal resemble the overall structure, either exactly or in a slightly varied form. This similarity appears across different scales. • **Recursion**: Fractals are formed through a repeating process where a simple rule or equation is applied repeatedly. This

	4. How does this activity reinforce what you learned from the examples in nature and art?	recursive repetition generates increasingly complex patterns over time. • **Infinite Complexity**: Fractals can be zoomed into endlessly, with each level revealing more detail without reaching a final, simple form. **Fractals in Nature and Art** Fractals are extremely common in nature because many natural processes grow through repeated patterns. Examples include trees, rivers, coastlines, mountains, clouds, seashells, and hurricanes. In these systems, branching or spreading structures often follow fractal-like rules, producing similar shapes at different scales. In art, fractals appear in both traditional inspiration and digital creation. Artists use fractal patterns to create visually complex and aesthetically appealing designs that reflect infinity, symmetry, and natural order. Computer-generated fractal art, such as images based on the Mandelbrot Set, demonstrates how mathematical recursion can produce intricate and beautiful visual forms. **Applications of Fractals** Fractals have important applications across different fields. In mathematics and computer science, they are used to model complex systems and generate detailed images using simple equations, such as the Mandelbrot Set. In nature and environmental science, fractal models help describe irregular structures like coastlines, mountain ranges, and cloud formations. In technology, fractals are used in image compression, computer graphics, and digital animation to create realistic natural landscapes. In medicine, fractal geometry helps analyze structures such as blood vessels and lung tissues, which also follow branching patterns. Overall, fractals provide a powerful way to understand, simulate, and represent complex systems in both science and art.

	B.3. Developing Mastery
	Activity B.3. PRACTICING LEARNED SKILLS Before facilitating this activity, ensure that all learners have a copy of the LAS. **Part I. Pattern Analysis** 1. Direct learners to examine each pattern carefully before classifying it. 2. Remind learners to use the defining characteristics of fractals, Fibonacci sequences, and the Golden Ratio when making classifications. 3. Encourage learners to justify every answer using observable features rather than guessing. 4. Circulate around the classroom and ask probing questions such as: 5. What feature helped identify the pattern? 6. Does the pattern repeat at different scales? 7. Is there evidence of a numerical sequence or proportional relationship? 8. Facilitate a brief discussion after completion and invite learners to explain their reasoning. 9. Clarify misconceptions regarding the differences among fractals, Fibonacci patterns, and Golden Ratio designs. **Part II. Manual Fractal Construction** 1. Organize learners into pairs. 2. Verify that all required materials are available before the activity begins. 3. Explain the concept of iterations and demonstrate how a simple fractal evolves from Iteration 0 to subsequent iterations. 4. Encourage learners to plan the fractal design before drawing. 5. Monitor the construction process and ensure that each iteration follows a consistent rule. 6. Ask guiding questions such as: a. What rule is being repeated in every iteration? b. How does each iteration relate to the previous one? c. Which feature demonstrates self-similarity? 7. Encourage neatness, accuracy, and creativity in presenting the fractal. 8. Review the written descriptions and ensure that learners explain the mathematical features of their designs. **Part III. Digital Fractal Creation** 1. Ensure that learners have access to the required devices and applications. 2. Demonstrate basic navigation of the selected software or online fractal generator. 3. Explain the expectations for the infographic output and the required descriptions. 4. Encourage experimentation with different parameters and design choices. 5. Monitor learner progress and provide technical assistance when needed. 6. Ask learners to compare the advantages of digital construction with manual construction. 7. Remind learners to save their outputs in the required format. 8. During presentations, encourage learners to explain: a. The rule used to generate the fractal.

<table>
<tr><td></td><td colspan="2">b. The appearance of self-similarity and recursion.
c. Connections to nature, art, or design</td></tr>
<tr><td rowspan="2">C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: CREATING FRACTAL PATTERN
Instructions:
1. Work individually or in pairs to design and create an artwork inspired by any of the following mathematical concepts:
a. Geometric symmetries
b. Tessellations
c. Frieze patterns
d. Fibonacci sequence
e. Golden Ratio
f. Fractals
2. Choose your prefer medium (drawing, painting, digital design, collage, sculpture, etc.) and plan a layout that clearly shows the chosen mathematical concept.
3. Write short description (2–3 paragraphs) explaining the mathematics behind their artwork, including:
a. The concept(s) applied
b. How the concept is represented in the artwork
c. Connections to nature, architecture, or design
4. Submit your artwork and written explanation for display in a class exhibit through gallery walk.

Processing Questions:
1. Which mathematical concept did you choose for your artwork, and why?
2. How does your artwork demonstrate the chosen concept (symmetry, tessellation, fractal, Fibonacci sequence, or Golden Ratio)?
3. How is your concept inspired by nature, art, or design?
4. What challenges did you encounter in applying mathematics creatively?
5. How does observing your classmates' artworks help you understand the different ways mathematics can be applied in creative contexts?</td><td>Activity C.1.
MAKING REAL-WORLD CONNECTIONS: CREATING FRACTAL PATTERN
Before the Activity
1. Prepare art materials and ensure the availability of digital tools, if applicable.
2. Display sample artworks that demonstrate mathematical concepts.
3. Review the characteristics of geometric symmetries, tessellations, frieze patterns, Fibonacci sequence, Golden Ratio, and fractals.
4. Explain the expectations for the artwork, written explanation, and gallery walk.
5. Present the assessment criteria, emphasizing mathematical accuracy, creativity, craftsmanship, and quality of explanation.

During the Activity
Planning Stage
1. Allow learners to work individually or in pairs.
2. Encourage learners to select a mathematical concept that interests them.
3. Guide learners in brainstorming possible themes inspired by nature, architecture, technology, culture, or everyday objects.
4. Ask guiding questions such as:
• Which mathematical concept will be featured in the artwork?
• How will the concept be represented visually?
• What real-world examples inspired the design?
5. Review initial plans before learners begin creating their artwork.

Creation Stage
1. Monitor learners as they construct their artwork.</td></tr>
</table>

		2. Encourage accurate application of the selected mathematical concept. 3. Provide assistance in using tools and materials when necessary. 4. Ask learners to explain the mathematical ideas being incorporated into their designs. 5. Encourage creativity while maintaining mathematical integrity. Writing Stage 1. Remind learners that the written explanation should clearly connect mathematics and art. 2. Encourage inclusion of: 3. The mathematical concept applied 4. How the concept appears in the artwork 5. Real-world connections to nature, architecture, technology, or design 6. Review drafts and provide feedback on clarity and mathematical accuracy. **Gallery Walk and Presentation** 1. Display all artworks around the classroom. 2. Organize a gallery walk where learners observe the work of classmates. 3. Encourage learners to read the accompanying explanations while examining each artwork. 4. Provide observation sheets or guide questions for recording insights during the gallery walk. 5. Invite selected learners or pairs to briefly explain their artwork to the class. 6. Encourage respectful feedback and appreciation of diverse creative approaches.
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Instructions**: To synthesize the discussion, answer the following questions:	**Activity C.2. WRAPPING UP THE LESSON** To facilitate the synthesis, facilitate the provided activity.

	1. How would you define a fractal in your own words using the terms self-similarity, recursion, and infinite complexity? 2. Which characteristic of fractals (self-similarity, recursion, or infinite complexity) is most evident in natural objects? Why? 3. List at least three examples of fractals observed in nature and three examples in human-made designs or artworks. 4. What common features do natural and artistic fractals share? 5. How do fractal patterns in nature inspire human-made designs or artworks? 6. What similarities and differences do you notice between fractals in natural objects and man-made designs? 7. Based on your observations, what general principle can you state about the use of fractal patterns in both nature and art?	
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a **copy** of the LAS. **Part I. Key Concepts of Fractals** 1. The teacher distributes the learning activity sheets to the learners. 2. Read the directions aloud: 3. Inform learners that the test will be answered individually. 4. Monitor the class while learners are answering to ensure that they are working independently. **Part II. Generating Fractals** 1. Begin by presenting sample fractal images and guiding learners to observe patterns of repetition and self-similarity. 2. Clearly explain the task: learners will design a small fractal using geometric shapes or nature-inspired patterns, ensuring at least 2–3 levels of iteration, and write a short explanation describing the rule or pattern used, how repetition or recursion is shown, and how self-similarity is evident. 3. Model a simple example to support understanding, then allow learners to work independently while providing guidance, especially for those who need support through templates or step-by-step assistance. 4. Encourage advanced learners to create more complex or digital designs. 5. After completion, select some learners to present their work and explain their patterns, then facilitate a brief discussion to reinforce key concepts. 6. Assess outputs using a rubric focusing on understanding, application, creativity, and clarity of explanation, and conclude by emphasizing the presence of fractals in nature, art, and technology.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. EXTENDING AND REINFORCING LEARNING	

	Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Ensure the availability of art materials, graph paper, rulers, coloring materials, and digital devices when applicable. 2. Review the characteristics of fractals, including self-similarity, iteration, recursion, and scaling. 3. Explain the purpose of differentiated tasks and the expectations for each level. 4. Assign learners to appropriate levels based on readiness, performance, or teacher discretion. 5. Explain the instructions and expected outputs for each level. 6. Encourage learners to work independently while seeking assistance when necessary. 7. Monitor progress and provide timely support. 8. Ask guiding questions that promote observation, analysis, and reflection. Encourage learners to connect mathematical concepts with visual patterns and real-world examples. 9. Provide feedback throughout the activity to improve understanding and performance.
V. ASSESSMENT	**To evaluate learners' success in attaining the intended learning competencies, the assessment tools and strategies provided on the link in the table of contents can be utilized to measure understanding, skills, and application of concepts**
VI. REFLECTION	**To assess and evaluate the effectiveness of the instruction, as well as to identify challenges and plan for improvements in this unit, teachers are encouraged to answer the reflective questions provided in the link indicated in the table of contents**

UNIT 3. INTRODUCTION TO MATRICES

I. LEARNING GOALS	
Content Standard	The learners demonstrate knowledge and understanding of fundamental concepts of matrices and basic operations.
Performance Standard	By the end of the quarter, the learners are able to model real-world situations using matrices and matrix operations. They are also able to formulate linear programming models and solve them using either the graphical or simplex method.
Learning Competencies	The learners… 1. illustrate matrix and its components (e.g., row, column, dimensions, and order of a matrix); 2. perform matrix operations (addition, subtraction, scalar multiplication, multiplication, transposition); 3. apply properties of matrix operations (commutative, associative, distributive, zero matrix, identity matrix, etc.); 4. represent systems of linear equations using matrices. 5. apply row operations to solve a system of linear equations. 6. define the inverse of a matrix 7. calculate the inverse of a matrix using row operations 8. use row operations to determine if a system of linear equations is independent (unique solution), dependent consistent (infinitely many solutions), or inconsistent (no solution) 9. *compute the determinant of a 2×2 and a 3×3 matrix;* 10. use the determinant to test matrix invertibility; 11. use Cramer's rule to solve an independent system of linear equations 2×2 and 3×3 matrices
II. REFERENCES and MATERIALS	
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Website and Online Article	• Elaine. (2015, July 22). *Section 16.3: Matrix Solution of a System of Equations. The Nature of Mathematics – 13th Edition*. Retrieved from https://mathnature.com/essential-ideas-16-3/ • Khan Academy. *Cramer's Rule*. • Khan Academy. *Introduction to Matrices*. • Nicholson, W. K. (n.d.). *1.1: Solutions and Elementary Operations*. In *Linear Algebra with Applications (Nicholson)*. Mathematics • LibreTexts. Retrieved March 12, 2026, from https://math.libretexts.org/Bookshelves/Linear_Algebra/Linear_Algebra_with_Applications_%28Nicholson%29/01%3A_Systems_of_Linear_Equations/1.01%3A_Solutions_and_Elementary_Operations

Video Lesson	• https://www.youtube.com/watch?v=F37rd-BpVOo&t=5s • https://www.youtube.com/watch?v=JMjbPh1Mjn8 • https://www.youtube.com/watch?v=ZCmVpGv6_1g • https://www.youtube.com/watch?v=7jb_AO_hRc8 • https://www.youtube.com/watch?v=4lHyTQH1iS8 • https://www.youtube.com/watch?v=o6tGHLkZvVM • https://www.youtube.com/watch?v=g_Rz94DXvNo
Materials	1. Instructional Materials/ Slide deck presentation 2. Printed worksheets 3. Pen or pencils 4. Notebook or papers 5. Whiteboard/ marker or projector 6. Role cards/ labels 7. Reference sheet (if needed)
EdTech Tool	1. Scientific Calculator (optional)
Use of Generative AI	In preparing this Learning Exemplar (LE), the author(s) used generative artificial intelligence (AI) tools to assist in organizing ideas, developing sample activities, and generating formative assessment items. The author(s) carefully reviewed, validated, and finalized all content. The author(s) take full responsibility for the content of this LE.

III. CONTENT	**Lesson 3.1. FUNDAMENTAL CONCEPTS AND OPERATIONS OF MATRICES**	
IV. OBJECTIVES	**At the end of the lesson, the learners are expected to:** **1. define a matrix and identify its components, including elements, rows, columns, and dimensions;** **2. calculate the sum, difference, and product of matrices including scalar multiplication;** **3. apply the rule of transposition to any given matrix; and** 4. **demonstrate understanding of matrix operations in real-life problems.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATAION GUIDE)**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge (10 mins)***	
	Activity A.1. LEVELING LEARNER READINESS **Option 1: Matrix Identification Task** **Instruction:** Study the matrix given. Identify the number of rows and columns, determine the dimension, and identify the element of the given matrix. $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ **Processing Questions:** 1. How many rows does the matrix have? 2. How many columns does it have? 3. What is the dimension of the matrix?	**Activity A.1. LEVELING LEARNER READINESS** **Option 1: Matrix Identification Task** 1. Use read across, drop down strategy. Tell the learners that rows are like floors in the classroom – they go "across" horizontally and columns are like the pillars holding up the classroom – they go "down" vertically. 2. For element, use address analogy. Tell the learners that the first number is the row number (where you land), and the second number is the column number (how far you move right). 3. For processing questions, you may apply "**Think-Pair-Share"** strategy.

4. Identify element a_{12}.
5. If a third row [1, 6] is added to the Matrix A, what would be the new dimension of the matrix?

Option 2: Data Table Recall

Instruction: Study the table below.

LEARNER	Math Score	Science Score
Ana	85	90
Ben	88	86
Cara	92	89

Processing Questions:

1. What if we consider each LEARNER as a row in a matrix? How many rows would the matrix have?
2. What if each subject is represented as a column in a matrix? How many columns would the matrix have?
3. What if we convert this table into a matrix with LEARNERs as rows and subjects as columns? What would be the dimension of the matrix?
4. What if a new subject, say “English Score,” is added as another column? How would this change the matrix dimensions?

a. **Think:** Learners will answer the 5 questions individually.
b. **Pair**: They turn to a partner and compare answers. If they have a different answer they must "defend" their choice using the row/column rule.
c . **Share:** Call on pairs to provide the answers. If a pair got a wrong answer initially but corrected it during the "Pair" phase, ask them to explain *why* they changed their mind—this clarifies common misconceptions for the whole class.

Option 2: Data Table Recall

1. Start by asking the class, *"Why do we use tables?"* (Answer: To organize information). Explain that a Matrix is just a table stripped of its labels (names and headers) to make it easier for computers and mathematicians to calculate.
2. Tell the learners to count the names vertically for rows and the subjects horizontally for columns. Remind them that "order" is the same as "dimension." It is always LEARNERs (rows) × subjects (columns).
3. Draw an empty column to the right of the table on the board. Label it "English." What if a new subject, say “English Score,” is added as another column? How would this change the matrix dimensions?

A.2. Establishing the Purpose of the Lesson

Activity A. 2. APPRECIATING LESSON RELEVANCE

Option 1: Investigating Population Trends
Instructions:
1. Study the table carefully.
2. Divide the class into groups. Work with your group to analyze the data.
3. Answer the questions and explain your reasoning.

Population in Millions	2020	2024
CALABARZON	16.20	16.93

Activity A. 2. APPRECIATING LESSON RELEVANCE

Option 1: Investigating Population Trends

1. Introduce the concept that census data is the "blueprint" for the nation. Ask the class: "If you were a city planner, what would you need to build if your city’s population suddenly doubled?" (Lead them to answers like hospitals, schools, transportation, and housing).
2. Before answering the questions, ensure the learners can accurately read the table.
   a. Use a pointer to demonstrate that the Rows represent Regions and Columns represent Years.

NCR	13.48	14.00
Central Luzon	12.42	12.99

Processing Questions:
1. What was the population of the Philippines in 2020?
2. Between which pair of consecutive census years did the Philippines experience the biggest growth? Explain.
3. Why do you think increased population data is important for economic planning in the Philippines?

Option 2: Investigating Population Trends
Instructions:
1. Divide the class into groups.
2. Observe the population data in the table.
3. Answer the questions individually first.
4. Discuss your answers with your group.
5. Explain how the data shows population trends.

Country (Population in Millions)	2010	2015	2020
India	1,234	1,311	1,380
China	1,341	1,371	1,412
United States	309	321	331

Processing Questions:
1. Which country had the largest population in 2020?
2. Which country had the smallest population in 2010?
3. How much did the population of India increase from 2010 to 2020?
4. Which country experienced the largest population growth overall? Explain how you know.
5. If this data were written as a matrix, how could matrices help compare population growth more efficiently?

b. Clarify that the numbers are in "millions," so 16.20 is 16,200,000.
3. Divide the learners into groups to work through the three guide questions.
4. Bring the class back together to compare answers. Use the board to visualize the growth—if you draw this data as a chart, the trends become immediate and obvious.

You may also use this strategy:
The "Data Investigator" Jigsaw - this ensures every learner is responsible for the analysis of specific regions.
1. Break the table into rows. Assign one group to be the "CALABARZON Experts," one to be "NCR Experts," and one to be "Central Luzon Experts."
2. Each group calculates the exact population increase for their region. Then, have them create a "Growth Fact Sheet" for their assigned region.
3. Bring the experts together to discuss which region is growing the fastest. This makes the activity more personal and competitive in a healthy, engaging way.

Option 2: Investigating Population Trends
1. Tell the learners to form groups.
2. Distribute the table and process questions.
3. Have the learners work through questions 1–4 independently to ensure baseline comprehension.
4. Groups discuss their answers, specifically comparing their methods for calculating growth.
5. Address question 5 as a whole class. This is the moment for applying matrices to data.

B.1. Presenting Examples

B. Instituting New Knowledge	**Activity B.1. EXPLORING KEY CONCEPTS** **Understanding Key Concepts** A **matrix** is a rectangular arrangement of numbers, symbols, or expressions in rows and columns. It is used to organize data and can help solve real-life problems in mathematics, science, and other fields. **Components of a Matrix** • **Element –** each individual number or symbol in a matrix. • **Rows –** horizontal lines of elements. • **Columns –** vertical lines of elements. • **Dimension –** indicates how many rows and columns the matrix has. The **dimension** of a matrix is written as $\boldsymbol{i \times j}$, where $\boldsymbol{i}$ is the number of rows and $\boldsymbol{j}$ is the number of columns. **Examples:** 1) 2 × 2 Matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ • Elements: 1, 2, 3, 4 • Rows: [1, 2], [3, 4] • Columns: [1, 3], [2, 4] • Dimension: 2 × 2 (2 rows, 2 columns) 2) 3 x 3 Matrix $B = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \\ 11 & 12 & 13 \end{bmatrix}$ • Elements: 5, 6, 7, 8, 9, 10, 11, 12, 13 • Rows: [5, 6, 7], [8, 9, 10], [11, 12, 13] • Columns: [5, 8, 11], [6, 9, 12], [7, 10, 13] • Dimension: 3 × 3 (3 rows, 3 columns) 3) 2 x 3 Matrix $C = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 9 & 6 \end{bmatrix}$ • Elements: 1, 2, 4, 6, 7, 9 • Rows: [2, 7. 1], [4, 9, 6] • Columns: [2, 4], [7, 9], [1, 6]	**Activity B.1 EXPLORING KEY CONCEPTS** This is the part of the lesson where the teacher will discuss the key concepts of matrix – definition and its components. **Strategy:** Explicit teaching through systematic approach where you clearly explain the concept, models examples, and guides learners through practice before allowing them to work independently. You must ensure that learners clearly understand the definitions, structure, and operations of matrices.

	• Dimension: 2 × 3 (2 rows, 3 columns) **Option 1: Gallery Walk** **Instructions:** 1. Walk around the classroom to observe the posted matrices. Each station has different matrices. 2. At each station, answer the following questions on your notebook. a. How many rows are there? b. How many columns are there? c. What are the elements of the matrix? d. What is the dimension of the matrix? 3. Look at the differences between the matrices. Notice how the number of rows and columns affects the total number of elements. 4. After visiting all stations, form small groups. Discuss what you noticed about rows, columns, and dimension in each matrix. 5. Present your interesting observation to the class. **Example Matrices for the Activity:** Station 1: $A = [10 \quad 20]$ Station 2: $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Station 3: $C = \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$ Station 4: $D = \begin{bmatrix} -1 & 4 & 3 \\ 0 & 10 & -5 \end{bmatrix}$ Station 5: $E = \begin{bmatrix} 0 & 1 \\ 7 & 5 \\ 4 & 8 \end{bmatrix}$ Station 6: $F = \begin{bmatrix} -5 & 0 & 5 \\ 3 & 1 & 2 \\ 6 & 0 & -2 \end{bmatrix}$ **Processing Questions:** 1. How many rows are in the matrix? 2. How many columns are in the matrix? 3. What are the elements of the matrix? 4. What is the dimension of the matrix? 5. How does the number of rows and columns affect the total number of elements?	**Option 1: Gallery Walk** **To facilitate this activity:** 1. Prepare stations around the classroom with different matrices posted around the classroom. 2. Ensure each station has a unique matrix for learners to observe. 3. Tell the learners that they will walk around the classroom and analyze matrices at each station. 4. Tell the learners to walk around the classroom and visit each station then answer the following questions provided. 5. Instruct learners to notice differences between the matrices. Highlight how the number of rows and columns affects the total number of elements. 6. After completing the gallery walk, form small groups of 3–4 learners. Discuss their observations about rows, columns, elements, and dimensions. 7. Ask each group to share an interesting observation with the whole class. 8. Facilitate discussion, clarify misconceptions, and summarize key points about matrix and its components.

Option 2: Matrix Match Up Challenge
Instructions:
1. Form small groups, each group 3 – 5 learners.
2. Each group receives:
   - 5 matrix cards
   - 5 answer cards
3. For each matrix card, identify the elements, rows, columns and determine the dimension.
4. After completing the matching, discuss within your group why each match is correct. Identify differences between matrices.
5. Each group will present one of their matches and explain how they identified rows, columns, elements, and order.

Example Matrices for the Activity:

$\mathrm{A} = \begin{bmatrix} 1 & -3 \\ 7 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 6 \\ 5 & 1 & 7 \end{bmatrix}$ $C = \begin{bmatrix} 12 \\ -3 \end{bmatrix}$

$D = \begin{bmatrix} 4 & -1 \\ 0 & 3 \\ 5 & 2 \end{bmatrix}$ $E = \begin{bmatrix} 2 & -5 & 0 \\ 3 & 4 & -1 \\ 6 & -8 & 4 \end{bmatrix}$

Processing Questions:
1. How many rows are in the matrix card?
2. How many columns are in the matrix card?
3. What are the elements shown in the matrix card?
4. What is the dimension of the matrix card?
5. Why does the matched answer card correctly describe the matrix?

Option 2: Matrix Match Up Challenge
1. Ask learners to form small groups of 3–5 members.
2. Give each group 5 matrix cards and 5 answer cards.
3. Instruct groups to match the cards:
   - Examine each matrix card. Identify the elements, rows, and columns and determine the order.
4. Once all matches are complete, discuss within the group:
   - Why each match is correct and the difference between the matrices.
5. Ask the learners to present and explain how they identified the rows, columns, elements, and dimension.

Tip: Encourage learners to talk through their reasoning rather than guessing. Monitor groups and ask guiding questions if they get stuck.

B.2. Discussing the Concept

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Operations of Matrices

1. Matrix addition is performed by **adding the corresponding elements** of two matrices that have the **same dimension** (same number of rows and columns).

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

This is the part of the lesson where the teacher will discuss the operations of matrices.

Strategy: Explicit teaching through systematic approach where you clearly explain the concept, models examples, and guides learners through practice before allowing them to work independently. You must ensure that learners clearly understand

	Rule: If $A = [a_{ij}]$and $B = [b_{ij}]$ are matrices of the same dimension, then $$\boldsymbol{A + B = [a_{ij}] + [b_{ij}]}$$ **Take note:** Matrices must have the same dimensions to be added. **Examples:** 1. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix}$, compute A + B. $$A + B = \begin{bmatrix} 2+5 & 4+1 \\ 1+2 & 3+6 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 3 & 9 \end{bmatrix}$$ 2. If $C = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 0 \end{bmatrix}$ & $D = \begin{bmatrix} 8 & 3 & 5 \\ 1 & 6 & 4 \\ 0 & 2 & 7 \end{bmatrix}$, compute C + D. $$C + D = \begin{bmatrix} 1+8 & 0+3 & 2+5 \\ 3+1 & 1+6 & 4+4 \\ 5+0 & 7+2 & 0+7 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 7 \\ 4 & 7 & 8 \\ 5 & 9 & 7 \end{bmatrix}$$ **2. Matrix subtraction** is performed by **subtracting the corresponding elements** of two matrices with the **same dimension**. **Rule:** If $A = [a_{ij}]$and $B = [b_{ij}]$ are matrices of the same dimension, then $$\boldsymbol{A - B = [a_{ij}] - [b_{ij}]}$$ **Take note:** Matrices must have the same dimensions to be subtracted. **Examples:** 1. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix}$, compute A - B. $$A - B = \begin{bmatrix} 2-5 & 4-1 \\ 1-2 & 3-6 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -1 & -3 \end{bmatrix}$$ 2. If $C = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 0 \end{bmatrix}$ & $D = \begin{bmatrix} 8 & 3 & 5 \\ 1 & 6 & 4 \\ 0 & 2 & 7 \end{bmatrix}$, compute C - D $$C - D = \begin{bmatrix} 1-8 & 0-3 & 2-5 \\ 3-1 & 1-6 & 4-4 \\ 5-0 & 7-2 & 0-7 \end{bmatrix} = \begin{bmatrix} -7 & -3 & -3 \\ 2 & -5 & 0 \\ 5 & 5 & -7 \end{bmatrix}$$ **3. Matrix Multiplication** is the process of multiplying two matrices to get a new matrix. For two matrices A and B to be	the different operations and follow the steps/process in applying the operation.

multiplied, the **number of columns in** A must be equal to the **number of rows in** B.

Rule: C = A × B where each element c_{ij} of C is calculated as:

$$\mathbf{c_{ij} = (row\ i\ of\ A) \times (column\ j\ of\ B)}$$

Examples:

1. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix}$, compute A × B.

number of columns of matrix A = 2
number of rows of matrix B = 2

$$A \times B = \begin{bmatrix} (2 \times 5) + (4 \times 2) & (2 \times 1) + (4 \times 6) \\ (1 \times 5) + (3 \times 2) & (1 \times 1) + (3 \times 6) \end{bmatrix} = \begin{bmatrix} 18 & 26 \\ 11 & 19 \end{bmatrix}$$

2. If $C = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 9 & 6 \end{bmatrix}$ & $D = \begin{bmatrix} 4 & 1 \\ 0 & 3 \\ 5 & 2 \end{bmatrix}$, compute C × D.

number of columns of matrix C = 3
number of rows of matrix D = 3

$$C \times D = \begin{bmatrix} (2 \times 4) + (7 \times 0) + (1 \times 5) & (2 \times 1) + (7 \times 3) + (1 \times 2) \\ (4 \times 4) + (9 \times 0) + (6 \times 5) & (4 \times 1) + (9 \times 3) + (6 \times 2) \end{bmatrix}$$

$$C \times D = \begin{bmatrix} 13 & 25 \\ 46 & 43 \end{bmatrix}$$

4. Scalar Multiplication means multiplying every element of a matrix by a number (scalar).

Rule: If k is a number and A is matrix,

$$\boldsymbol{kA = [a_{ij}]}$$

Examples:

1. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$, compute 3A.

$$3A = \begin{bmatrix} 3(2) & 3(4) \\ 3(1) & 3(3) \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 3 & 9 \end{bmatrix}$$

2. If $B = \begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix}$, compute -2B.

$$-2B = \begin{bmatrix} -2(5) & -2(1) \\ -2(2) & -2(6) \end{bmatrix} = \begin{bmatrix} -10 & -2 \\ -4 & -12 \end{bmatrix}$$

5. Transpose of a Matrix is obtained by interchanging its rows and columns.

Rule: If matrix A is given, its transpose is written as:

$$\boldsymbol{A^T}$$

Examples:

1. If $A = \begin{bmatrix} 1 & -3 \\ 7 & 5 \end{bmatrix}$, find A^T.

$$A^T = \begin{bmatrix} 1 & 7 \\ -3 & 5 \end{bmatrix}$$

2. If $B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 0 \end{bmatrix}$, find B^T.

$$B^T = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 7 \\ 2 & 4 & 0 \end{bmatrix}$$

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS. Choose the activity that is most suited to the learners.

Option 1: Guided Practice and Independent Activity
Part I. Guided Practice

1. Group learners in pairs or small groups (3–4 members).
2. Provide matrices and a step-by-step guide for operations.
3. Guide learners through problems.
    - Highlight placement of elements in addition, multiplication, or transpose.
    - Encourage discussion within groups before writing final answers.
4. Walk around to clarify misconceptions and provide hints. Emphasize showing complete solutions for each operation.

Part II. Independent Learning

1. Instruct learners to work individually.
2. Remind them to:
    - Check matrix dimensions before performing any operation.
    - Apply the appropriate operation.
    - Show complete solutions and final answers in their notebooks.
3. Walk around to observe problem-solving and offer hints if necessary. Encourage learners to verify their results and cross-check intermediate steps.
4. Review selected problems as a class after completion, highlighting common errors and correct methods.

Option 2: The Collaborative Matrix Challenge

1. Tell the learners to form groups and instruct them to assign a role each member.

2. Give each group the matrix problems. Remind them to:
   - Check if the operation is possible (look at dimensions).
   - Solve the problem step by step.
   - Show all work and explain how each element was calculated.
3. Walk around to support and answer questions.
4. Let the learners present their work. Encourage other learners to ask questions.
5. Check all the answers to be presented and highlight key points
   - Always check dimensions.
   - Show all steps for clarity.
   - Collaboration helps understand the process.

C. Demonstrating Knowledge and Skills

C.1. Finding Practical Application (

Activity C.1. MAKING REAL-WORLD CONNECTIONS

Matrices in Action: Solving Real-World Problems
Instructions:
1. Divide the class into groups of 5 members.
2. Choose the task that best matches your group's talents.
3. Represent the data using matrices.
4. Perform the required matrix operations.
5. Prepare your final output (report, poster, presentation, or digital output).
6. Present your work to the class.

Scenario: Your class is helping the school administration analyze learner participation in three school programs:
   a. Sports
   b. Academic Clubs
   c. Arts Activities

Table A: Number of LEARNERs participating from three grade levels.

Grade Level	Sports	Academic Clubs	Arts
Grade 10	30	20	15
Grade 11	25	35	20
Grade 12	20	25	30

Table B: Number of LEARNERs planning to join next semester based on the survey

Grade Level	Sports	Academic Clubs	Arts

Activity C.1. MAKING REAL-WORLD CONNECTIONS

1. Introduce the activity by explaining how matrices help analyze real-world data.
2. Present the scenario and matrices to the class.
3. Allow LEARNERs to choose differentiated tasks according to their interests or strengths.
4. Provide guidance during computation and interpretation.
5. Monitor collaboration and ensure each member participates.
6. Facilitate group presentations and discussions.
7. Assess outputs using a performance rubric focusing on accuracy of matrix operations, clarity of interpretation, creativity, communication and collaboration.

Refer to Teachers' Note for the rubric.

Grade 10	5	3	2
Grade 11	4	6	3
Grade 12	3	2	5

Learners will choose one task based on their strengths or talents.

Group 1. Analytical Thinkers

Task: Data Analysis

Compute and interpret the following:

1. A + B: total number of participants next semester.
2. A – B: difference between current and projected participation.
3. 2A: if participation doubles during a school festival.
4. A^T: analyze participation by activity instead of grade level.

Output: A written analysis report explaining the meaning of each result.

Group 2. Creative Designers

Task: Visual Matrix Representation

Create a poster or infographic that:

1. Shows the matrices $A$and B.
2. Illustrates results of $A + B$and $2A$.
3. Uses colors, charts, or diagrams to explain matrix operations.

Output: A visual presentation explaining the matrices and their results.

Group 3. Communicators

Task: Real-Life Application Report

Create a short presentation or report explaining:

1. How matrices help organize school data.
2. The results of $A + B$and A^T.
3. How school administrators could use the information to improve programs.

Output: Presentation or short written report.

Group 4 – Technology Enthusiasts

Task: Digital Data Analysis

Using Excel, Google Sheets, or a calculator, perform:

1. A + B
2. A – B
3. 2A

	4. A^T Create a digital table or chart showing the results. Output: Digital spreadsheet or slide presentation. **Processing Questions**: 1. Why is it important to represent real-world data in matrix form? 2. How did your group decide which matrix operations to use for analyzing the data? 3. What challenges did you encounter when applying matrix operations to real-world data? 4. How did collaboration within your group help in solving the matrix problems? 5. What insights about LEARNER participation did your group discover through matrix analysis?	
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Option 1: Reflection** **Instruction:** After completing the activities on matrices and matrix operations, reflect on what you have learned. Answer the questions below in 2–3 sentences each. Your answers should summarize the important ideas from the lesson written on your notebook. **Questions:** 1. What is a matrix and what information can it represent in real-life situations? Give one example of how matrices can organize data. 2. Why is it important to know the order (rows and columns) of a matrix before performing operations such as addition or subtraction? 3. What did you notice about how elements are combined in matrix addition and subtraction? Explain the pattern you observed. 4. How is matrix multiplication different from matrix addition and subtraction? 5. What is one key idea you learned about matrices and their operations that you think will help you analyze data or solve problems in mathematics or real life?	**Activity C.2. WRAPPING UP THE LESSON** **Option 1: Reflection:** 1. Ask the learners to answer the questions in their notebook. For each question, answer in 2–3 sentences summarizing their learning. 2. Let the learners think about what they have learned, patterns they have observed, and how matrices can be applied in real life. 3. Remind them to reflect honestly, there are no wrong answers. 4. Walk around to observe understanding and provide prompts if learners are stuck. 5. Invite a few learners to share insights with the class to reinforce learning.

	Option 2: Concept Map **Instructions:** 1. Review the concepts discussed about matrices, including their components and operations. 2. Write "Matrices" at the center of your paper. This will be the main concept. 3. Draw branches from the word Matrices and write the following related concepts: Matrix Components and Matrix Operations. 4. Expand the ideas under: • Matrix Components – rows, columns, elements and dimensions • Matrix Operations – addition, subtraction, scalar multiplication, multiplication and transpose. 5. Write short explanation for each concept. 6. Use arrows or lines to connect related ideas to show their relationship. 7. Include an example on each matrix operation. 8. Share and discuss with your classmates and briefly explain your concept map. Compare how your ideas are organized.	**Option 2: Concept Map** 1. Tell learners that the concept map will help them summarize the lesson and visualize how matrices and their operations are related. 2. Instruct the learners to write "Matrix" at the center of the board or on a chart paper. 3. Ask learners to identify the components of a matrix. Branch out to operations such as addition, subtraction, scalar multiplication, multiplication, transpose. 4. Encourage them to use colors, arrows, and labels to show relationships clearly. 5. Discuss similarities and differences and highlight important connections.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3: Assessing Learning Outcomes Before facilitating this activity, ensure that all learners have a copy of the LAS. This part of the lesson is designed to measure how well learners have understood the key concepts of matrices and their operations. Learners will demonstrate their ability to: 1. Identify the components of a matrix (rows, columns, elements, dimension). 2. Perform matrix operations such as addition, subtraction, multiplication, scalar multiplication, and transpose.	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4: Extending and Reinforcing Learning Before facilitating this activity, ensure that all learners have a **copy** of the LAS. **Remedial Activity:** **To facilitate this activity:** 1. Identify learners who struggle with matrix concepts based on previous activities, quizzes, or observations. 2. Tell learners: "This activity will help you practice matrices step by step so you can understand and perform operations more confidently." 3. Depending on their level, learners can work individually or in pairs to receive guidance while solving problems.	

	4. Provide Simplified Problems: • Start with small matrices. • Focus on identifying rows, columns, and elements, determining the dimension of a matrix and performing basic operations (addition, subtraction, scalar multiplication, transpose). 5. Solve one problem together step by step on the board. Emphasize checking dimensions and showing all steps clearly. 6. Monitor and Support: Circulate around the room, giving hints and guiding questions instead of giving answers. 7. After learners complete the activity, review selected problems together. Highlight common mistakes and clarify key concepts. **Enrichment activity** 1. Identify learners who demonstrated mastery in guided and independent practice to participate in enrichment tasks. 2. Tell learners: "These activities will challenge you to apply your knowledge of matrices in more complex or real-life scenarios. The goal is to think critically and explore new ways to use matrices." 3. Depending on the activity, learners can work individually or in small groups to promote discussion and collaboration. 4. Provide advanced or more challenging problems. 5. Encourage exploration and creativity. Encourage learners to justify each step and explain their reasoning. 6. Have learners present interesting findings or solutions to the class.

III. CONTENT	**Lesson 3.2. PROPERTIES OF MATRIX OPERATIONS**	
IV. OBJECTIVES	**At the end of the lesson, the learners are expected to:** **1. identify and describe the properties of matrix operations such as commutative, associative, distributive, identity, and zero matrix properties;** **2. apply the properties of matrix operations (commutative, associative, distributive, zero matrix, identity matrix, etc.) to solve matrix expressions correctly; and** **3. appreciate the importance of mathematical rules in ensuring accuracy when performing matrix operations.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS **Option 1:** A school is preparing learning kits for LEARNERs. The supplies are recorded using matrices to organize the quantities distributed in two different weeks. $Week\ 1 = \begin{bmatrix} 120 & 80 \\ 100 & 60 \end{bmatrix}$ $Week\ 2 = \begin{bmatrix} 90 & 70 \\ 110 & 50 \end{bmatrix}$ **Instructions:**	**Activity A.1. LEVELING LEARNER READINESS** **Option 1** **To facilitate this activity:** 1. Ask learners to recall the rules for matrix addition and subtraction before starting the activity. 2. Present the context of school supply distribution to help LEARNERs see how matrices organize real data. 3. Allow learners to work in pairs. 4. Encourage learners to explain why matrices must have the same dimensions for addition and subtraction.

	1. Study the matrices below showing the number of school supplies distributed to LEARNERs in Week 1 and Week 2. 2. Perform the required matrix operations. 3. Show your complete solution in your notebook. 4. Work with a partner and compare answers after solving. **Processing Questions:** 1. What operation must be performed to determine the total number of supplies distributed across Week 1 and Week 2? 2. What are the total supplies distributed in two weeks? 3. What is the difference in distribution between week 1 and week 2? 4. Why is organization of data important in schools? **Option 2:** A school distributed sports equipment to two grade levels (Grade 9 and Grade 10) over two weeks. The data is recorded in matrices: $\text{Week 1} = \begin{bmatrix} 50 & 30 \\ 40 & 20 \end{bmatrix} \text{Week 2} = \begin{bmatrix} 60 & 25 \\ 35 & 30 \end{bmatrix}$ *(Rows represent Grade 9 and Grade 10; columns represent basketballs and volleyballs.)* **Instructions:** 1. Study the matrices showing sports equipment distribution in Week 1 and Week 2. 2. Perform matrix addition and subtraction to analyze the data. 3. Answer the processing questions in one sentence each. 4. Work with a partner to compare and discuss your answers. **Processing Questions:** 1. What operation must be performed to find the total equipment distributed in two weeks? 2. What are the total basketballs and volleyballs distributed in two weeks? 3. What is the difference in distribution between Week 1 and Week 2? 4. Why is organizing sports equipment data important in schools?	5. After solving, facilitate a short discussion to answer processing questions. **Option 2:** **To facilitate this activity:** 1. Ask learners to recall the rules for matrix addition and subtraction before starting the activity. 2. Present the context of school supply distribution to help learners see how matrices organize real data. 3. Allow learners to work in pairs. 4. Encourage learners to explain why matrices must have the same dimensions for addition and subtraction. 5. After solving, facilitate a short discussion to answer processing questions.
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. APPRECIATING LESSON RELEVANCE	**Activity A. 2: APPRECIATING LESSON RELEVANCE**

	Imagine you are managing two small delivery services. Each matrix represents the number of packages delivered to different neighborhoods over two days. You want to combine, scale, or adjust these numbers efficiently. How can you be sure your calculations are correct every time? Observe the matrices displayed on the board or on your worksheet: $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **Instructions:** 1. Perform the following operations in pairs/ small groups. • Add $A + C$ and $C + A$ • Multiply $A \times B$and $B \times A$ • Multiply $2 \times (A + C)$ 2. Answer the processing questions. 3. Share observations with the class. **Processing Questions:** 1. Do the results change if the order of addition or multiplication changes? 2. What happens when you multiply a matrix by zero or the identity matrix? 3. What patterns or rules do you notice?	1. Start with a story: "You need to combine deliveries from two branches. What happens if we don't follow proper rules?" 2. Guide discovery: Allow learners to perform operations and observe patterns. Ask: • Does the order matter for addition? • Which matrix doesn't change the result when added? • Which matrix doesn't change the result when multiplied? 3. After learners notice patterns, formally name the properties: • Commutative property of addition • Associative property • Distributive property • Zero matrix property • Identity matrix property 4. Emphasize that knowing these properties helps simplify calculations and avoid mistakes in real-life data handling.
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS Suppose you are organizing the production schedule for a small bakery. Each matrix represents the number of breads, cakes, and pastries made on different days in different ovens. Understanding matrix operations and their properties will help you efficiently combine schedules, adjust outputs, and predict totals. **Instructions:** 1. Work in groups 3-4 members. Each group is assigned matrices: $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2. Perform the following operations and answer questions: a. Compute $A + B$and $B + A$. Are the results the same?	**Activity B.1. EXPLORING KEY CONCEPTS** 1. Prepare matrices on worksheets or the board. 2. Ensure learners have paper, pencils, and calculators (optional). Let learners calculate first, then discuss patterns. 3. Encourage them to label which property each result demonstrates. 4. Guided Discussion: • Highlight commutative property (order doesn't matter in addition). • Highlight associative property (grouping doesn't matter in addition/multiplication). • Highlight distributive property (scalars distribute over addition).

	b. Compute $(A + B) + C$and $A + (B + C)$. Are the results the same? c. Compute $2(A + B)$ and 2A + 2B. Compare the results. d. Compute $A + C$ and $A \times I$. Discuss how the zero matrix and identity matrix affect results. **Processing Questions:** 1. Which property makes addition easier when combining multiple matrices? 2. How can knowing these properties save time and avoid mistakes in real-world tasks? 3. Can you think of other real-life examples where matrix properties are useful?	• Highlight zero and identity matrix properties (don't change the original matrix).
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Properties of Matrices** **Matrix Addition** **1. Commutative Property of Addition -** the order of addition does not affect the result. **A + B = B + A** Example: $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$ $A + B = \begin{bmatrix} 2+5 & 3+1 \\ 1+0 & 4+2 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 1 & 6 \end{bmatrix}$ $B + A = \begin{bmatrix} 5+2 & 1+3 \\ 0+1 & 2+4 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 1 & 6 \end{bmatrix}$ **2. Associative Property of Addition -** how matrices are grouped in addition does not affect the result. **(A + B) + C =A+ (B + C)** Example: $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** In this stage, the facilitator leads a discussion on the properties of matrix operations, including commutative, associative, distributive, zero matrix, and identity matrix properties. Learners are guided to examine examples, observe patterns, and explain why certain properties hold or do not hold for matrices. The discussion emphasizes understanding rather than memorization, helping learners connect the properties to actual computations and real-life applications.

$$(A + B) + C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 3 & 7 \end{bmatrix}$$

3. Distributive Property - a scalar or matrix can be distributed over addition.

k(A + B) = kA + kB

Example:
Let $k = 2$

$$A + B = \begin{bmatrix} 7 & 4 \\ 1 & 6 \end{bmatrix}$$

$$2(A + B) = 2\begin{bmatrix} 7 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ 2 & 12 \end{bmatrix}$$

$$2A + 2B = 2\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + 2\begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 10 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 8 \\ 2 & 12 \end{bmatrix}$$

4. Zero Matrix Property - adding a zero matrix does not change the original matrix.

A + 0 = A

Example:

$$A + 0 = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$$

Matrix multiplication is different from addition because the order of multiplication matters, and not all the rules from addition apply.

1. Non-Commutative Property - the order of multiplication matters; $\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$ in most cases.

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} (1)(3) + (2)(1) & (1)(0) + (2)(2) \\ (0)(3) + (1)(1) & (0)(0) + (1)(2) \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} (3)(1) + (0)(0) & (3)(2) + (0)(1) \\ (1)(1) + (2)(0) & (1)(2) + (2)(1) \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$$

2. Associative Property - grouping does not affect the result in multiplication.

$$\mathbf{(A \times B) \times C = A \times (B \times C)}$$

Example:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

Compute $(A \times B) \times C$ and $A \times (B \times C)$

- Both will result in the same matrix:

$$(A \times B) \times C = A \times (B \times C) = \begin{bmatrix} 1 & 3 \\ 5 & 8 \end{bmatrix}$$

3. Distributive Property - multiplication distributes over addition.

$$\mathbf{A \times (B + C) = A \times B + A \times C}$$

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 3 & 1 \\ 3 & 3 \end{bmatrix}$$

$$A \times (B + C) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 3 & 3 \end{bmatrix}$$

$$A \times B + A \times C = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 3 & 3 \end{bmatrix}$$

4. Identity Matrix Property - multiplying a matrix by an identity matrix does not change the matrix.

$$\mathbf{A \times I = I \times A = A}$$

Example:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$A \times I = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, I \times A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

5. Multiplication by Zero Matrix - multiplying any matrix by a zero matrix results in a zero matrix.

$$\boldsymbol{A} \times \mathbf{0} = \mathbf{0}, \qquad \mathbf{0} \times \boldsymbol{A} = \mathbf{0}$$

Example:

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

Transpose of a Matrix
Properties

1. Transpose of a Transpose - taking the transpose twice returns the original matrix.

$$(\mathbf{A^T})^\mathbf{T} = \mathbf{A}$$

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, (A^T)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$$

2. Transpose of a Sum - the transpose of a sum equals the sum of the transposes.

$$(\mathbf{A + B})^\mathbf{T} = \mathbf{A^T + B^T}$$

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 \\ 6 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 7 \\ 9 & 5 \end{bmatrix}, (A + B)^T = \begin{bmatrix} 1 & 9 \\ 7 & 5 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 6 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 9 \\ 7 & 5 \end{bmatrix}$$

3. Transpose of a Scalar Multiple - transpose of a matrix multiplied by a scalar equals the scalar multiplied by the transpose.

$$(\mathbf{kA})^\mathbf{T} = \mathbf{kA^T}$$

Example:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, k = 3$$

$$3A = \begin{bmatrix} 6 & 9 \\ 3 & 12 \end{bmatrix}, (3A)^T = \begin{bmatrix} 6 & 3 \\ 9 & 12 \end{bmatrix}$$

$$3A^T = 3\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 9 & 12 \end{bmatrix}$$

4. Transpose of a Product - the transpose of a product reverses the order of multiplication.

$$(\mathbf{AB})^{\mathbf{T}} = \mathbf{B}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}$$

Example:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \cdot 3 + 2 \cdot 1 & 1 \cdot 0 + 2 \cdot 2 \\ 0 \cdot 3 + 1 \cdot 1 & 0 \cdot 0 + 1 \cdot 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

$$\left(AB\right)^T = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}, B^T A^T = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

5. Symmetric Matrices - A matrix is symmetric if it is equal to its transpose.

$$\mathbf{A} = \mathbf{A}^{\mathbf{T}}$$

Example:

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix},\ A^T = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

Activity B.3: Practicing Learned Skills

Before facilitating this activity, ensure that all learners have a **copy** of the LAS. Choose the activity that is most suited to the learners.

Option 1: Guided Practice and Independent Learning

Part I. Guided Practice

1. Guide learners in recalling the properties of matrix operations before starting.
2. Monitor group work and provide hints when necessary.
3. Ask probing questions to help learners justify their answers.
4. Clarify misconceptions immediately based on learners' responses

Part II. Independent Learning

1. Allow learners to work independently while observing their progress.
2. Provide minimal guidance to encourage critical thinking.
3. Check selected outputs and give immediate feedback.

	4. Summarize key concepts after the activity. **Option 2: Collaborative Activity** 1. Tell the learners to form groups and instruct them to assign a role each member. 2. Give each group the problems. 3. Walk around to support and answer questions. 4. Let the learners present their work. Encourage other learners to ask questions. 5. Check all the answers to be presented. **Part I. Properties of Matrix Addition** 1. Introduce the real-life scenario: "How can café sales data be organized using matrices?" 2. Emphasize that order matters in addition. 3. Guide learners to perform operations carefully step by step, especially for matrix addition. 4. Ask questions: • "What happens when we add by identity or zero?" • "Does adding another matrix change the result?" 5. Highlight commutative, identity, zero, and distributive properties. **Part II. Properties of Matrix Multiplication** 1. Introduce the real-life scenario: *"How does changing oven schedules affect total production?"* 2. Emphasize that order matters in multiplication, unlike addition. 3. Guide learners to perform operations carefully step by step, especially for matrix products. 4. Ask questions: • "What happens when we multiply by identity or zero?" • "Does switching order change the result?" 5. Highlight non-commutative, identity, zero, and distributive properties. **Part III. Properties of Matrix Transpose** Begin with a question: *"How can flipping rows and columns help us analyze data differently?"* 1. Guide learners through each step carefully, highlighting how transpose affects rows and columns. 2. Encourage learners to check their work to see the patterns in properties. 3. Summarize all key transpose properties on the board: • $(A^T)^T = A$ • $(A + B)^T = A^T + B^T$ • $(kA)^T = kA^T$ • $(AB)^T = B^T A^T$
	C.1. Finding Practical Application

<table>
<tr>
<td>C. Demonstrating Knowledge and Skills</td>
<td>
Activity C.1. MAKING REAL-WORLD CONNECTIONS

Analyzing School Resource Data Using Matrix Properties
Goal: Apply properties of matrix operations (commutative, associative, distributive, identity, and zero matrix) to analyze and interpret real-world school data.
Role: You are a School Data Analyst tasked to help the administration organize and interpret school resource and usage data.
Audience: School administrators, teachers, and LEARNER leaders.

Situation: The school collected data on resource usage across departments. Your task is to analyze the data using matrix operations and justify results using matrix properties.

Product/Performance: You will produce one of the following outputs (differentiated choice):
• Infographic (visual learners)
• Written report (analytical learners)
• Oral presentation (verbal learners)
• Spreadsheet/Matrix solution (logical learners)

Standard: Rubric for Assessing Performance
• To be presented by the teacher.

Your output must:
1. Show matrix computations
2. Identify properties used
3. Interpret results in real-life context

These are the following data:

Table 1: Weekly Resource Usage (Science and Math Departments)
<table>
<tr><th>Department</th><th>Computers (Morning)</th><th>Computers (Afternoon)</th></tr>
<tr><td>Science</td><td>20</td><td>15</td></tr>
<tr><td>Math</td><td>10</td><td>12</td></tr>
</table>
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<td>
Activity C.1. MAKING REAL-WORLD CONNECTIONS

Analyzing School Resource Data Using Matrix Properties
To facilitate this activity:
1. Introduce the real-life scenario to make the task meaningful.
2. Group the learners heterogeneously based on ability and strengths.
3. Allow learners to choose their output format (differentiation).
4. Guide learners in recalling matrix operations before starting.
5. Monitor group work and provide scaffolding where needed.
6. Ask probing questions to deepen understanding:
• “Why does this property hold?”
• “Does this apply to multiplication?”
7. Allow each group to present their output.
8. Provide feedback based on rubric criteria.
</td>
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Table 2: Week 2 Resource Usage

Department	Computers (Morning)	Computers (Afternoon)
Science	18	14
Math	12	10

Table 3: Computer Cost

Time Slot	Cost per Use (₱)
Morning	50
Afternoon	40

Instructions:

1. Form a group of 5 members.
2. Choose your preferred output format based on your strengths (poster, report, presentation, or spreadsheet).
3. Perform the following tasks:
    a. Compute total usage using $A + B$.
    b. Verify if matrix addition is commutative ($A + B = B + A$).
    c. Compute total cost using $(A + B)C$.
    d. Verify distributive property:
    $$(A + B)C = AC + BC$$
4. Analyze and interpret the results.
5. Present your findings clearly and justify which matrix properties were used.

Processing Questions:

1. Why does matrix addition satisfy the commutative property, but matrix multiplication does not always?
2. If a zero matrix is added to matrix A, what real-life meaning does it represent?
3. How does the distributive property help simplify large data computations?
4. In what situation might the identity matrix be useful in real-life data systems?
5. What would happen if matrix multiplication were commutative in real life?

C.2. Making Generalization

	Activity C.2. WRAPPING UP THE LESSON **Option 1: Reflective Questions** **Instructions:** 1. Review your work on matrix addition, multiplication, and transpose. 2. Reflect carefully and answer the following 5 questions in 2–3 sentences each. 3. Use your answers to formulate general rules for each type of operation. **Questions:** 1. When you added matrices in different orders or groupings, what patterns did you notice? How does this help you describe the rules of matrix addition? 2. When you multiplied matrices, did the order of multiplication affect the result? What did you observe about multiplying by the identity or zero matrix? 3. Did the distributive property hold for matrix multiplication? Explain how this property works in practical situations. 4. What happens when you transpose a matrix, the sum of matrices, or a product of matrices? What patterns can you observe? 5. Based on your observations, write general rules for matrix addition, multiplication, and transpose in your own words. **Option 2: Concept Map** **Instructions:** 1. Review all your activities and observations on matrix addition, multiplication, and transpose. 2. Using the concept map below, fill in the boxes with the correct properties and examples. 3. Connect each property to the operation it belongs to using arrows. 4. Write a brief description or example next to each property to show understanding.	**Activity C.2. WRAPPING UP THE LESSON** **Option 1: Reflective Questions** 1. Ask learners to reflect individually first, then discuss in small groups. 2. Encourage them to connect answers to real-life examples (sales, production, LEARNER scores). 3. Summarize the general rules on the board after discussion: • Addition: commutative, associative, zero matrix • Multiplication: non-commutative, distributive, identity, zero • Transpose: transpose of sum, product, scalar, double transpose. **Option 2: Concept Map** 1. Introduce the concept map: Explain that a concept map visually organizes properties of matrices and shows connections between concepts. 2. Ask learners to fill in missing boxes using their observations and activity results. 3. Encourage learners to add examples or mini-calculations next to each property. 4. Facilitate discussion of patterns: • Ask: *"Which operations are commutative?"*, *"Which are associative?"*, *"How does transpose relate to addition or multiplication?"* 5. Summarize and display a completed concept map for the class as a reference for future lessons.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES	

	Before facilitating this activity, ensure that all learners have a copy of the LAS. In this stage, learners demonstrate their understanding of matrix operations and their properties through this activity. They are evaluated on their ability to correctly apply, verify, and explain properties like commutative, associative, distributive, zero matrix, and identity matrix properties. The facilitator uses the results to determine mastery, identify learning gaps, and provide appropriate feedback or remediation.
	C.4. Additional Activities
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. **Remedial Activity** 1. Begin with a brief review of matrix operations before giving the activity. 2. Guide learners to focus on patterns, not only computation. 3. Provide step-by-step scaffolding if learners struggle with calculations. 4. Encourage learners to compare results (e.g., $A + B$ vs. $B + A$). 5. After the activity, ask learners to state the property in their own words. 6. Reinforce the lesson using visual examples or concept maps. **Enrichment Activity** 1. Present this activity as an extension task for advanced learners who quickly master the basic concepts. 2. Encourage LEARNERs to explain their reasoning, not only compute answers. 3. Ask probing questions such as: *a. Why does the property hold?* *b. Can this property apply to all matrices?* *c. How can this concept be used in real-life situations?* 4. Allow pair or group discussion to promote deeper mathematical thinking. 5. Conclude by asking learners to summarize the matrix properties they applied.

III. CONTENT	**Lesson 3.3. SOLVING SYSTEMS OF LINEAR EQUATIONS USING MATRICES AND ROW OPERATIONS**	
IV. OBJECTIVES	**At the end of this lesson the learners are expected to:** **1. represent a system of linear equations as an augmented matrix accurately;** **2. illustrate and perform all basic matrix row operations: row swapping, scalar multiplication, and row addition; and** **3. apply row operations to reduce matrices to their reduced row echelon form (RREF) and solve systems of linear equations.**	
V. PROCEDURES	**ACTIVITIES**	**ANNOTATION**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge (15 mins)***	
	Activity A.1. LEVELING LEARNERS READINESS	**Activity A.1. LEVELING LEARNERS READINESS**

	Instruction: Find a pair and then solve the following system of linear equations. $\{x + y = 5\ 2x + 3y = 11$ $\{x + y + z = 6\ 2x + 3y + z = 13\ x + 3y + 2z = 11$ **Processing Questions:** 1. How did you solve these systems? 2. How long did it take to solve each system? 3. What are the challenges that you encountered as the system increases its size?	The primary goal of this activity is to highlight the limitations of traditional algebraic methods (substitution and elimination) when applied to larger systems. By starting with a simple 2 x 2 and moving to a 3 x 3, learners will personally experience the increase in complexity, "bookkeeping" errors, and time required. This provides the pedagogical "hook" for matrices, positioning them not as a "new difficult topic," but as a tool for organization and efficiency. Encourage learners to work in pairs to solve both systems as quickly as possible using any method they remember from previous years. Walk around and notice which learners struggle with the 3 x 3 system. Many will likely get lost in the multiple steps of elimination or make a sign error. Use the Processing Questions to facilitate a class discussion. ● For Question 1: Most will say "elimination." ● For Question 3: Highlight words like "messy," "confusing," or "hard to keep track of variables." Use their struggle as a bridge: "What if I told you there is a way to solve these by stripping away the letters and just manipulating the numbers in a grid? That is what matrices are for."
	A.2. Establishing the Purpose of the Lesson (20 mins)	
	Activity A.2. APPRECIATING LESSON RELEVANCE Real-life examples of applying system of linear equations: 1. Business: Determining the number of products needed to meet revenue or resource constraints. The Scenario: A bakery sells Cupcakes for P 3.00 and Cookies for P2.00. Goal 1: They want to make exactly 50 items total today. Goal 2: They need to earn exactly P120 to cover their rent. The System: Let x be the number of cupcakes and y be the number of cookies. 1. $x + y = 50$ (Total items constraint) 2. $3x + 2y = 120$ (Revenue constraint)	**Activity A.2. APPRECIATING LESSON RELEVANCE** Explain that matrices provide a systematic, visual, and structured approach to representing and solving systems of linear equations. This serves as the Motivational Framework. After the struggle of the "Pair Activity," LEARNERs might feel that 3x3 systems are simply "more work." By citing Business, Engineering, and Computer Science, you are validating the utility of matrices in solving system of linear equations in professional fields. The teacher can state that: "*In our previous activity, we saw that as the number of variables increased, the chance of making an error also increases. In the professional world, we deal with hundreds of variables at once.*"

	2. Engineering: Solving electrical circuits, force balance problems, or chemical mixture ratios. The Scenario: Imagine a simple circuit with two loops. You need to find the current (I) flowing through two different resistors. According to physics, the sum of currents entering a junction must equal the sum leaving it, and the voltage drops around a loop must sum to the battery voltage. The System: If we have a battery and two resistors, the equations might look like this: 1. $I_1 + I_2 = 5$ (Total current from the battery) 2. $10I_1 - 5I_2 = 0$ (Voltage balance between two parallel branches) **Processing Question**: "If you had to solve a system with 5 or 10 variables, which method would be faster?"	Once learners are convinced of the *why*, you can immediately proceed with your lesson, where you show them the use of these matrices.
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS A system of linear equations can be represented by three matrices defined as follows: $AX = B$ Where: ● Matrix A – contains all the coefficients of the system ● Matrix X – contains all the variables that is in a form of a column matrix ● Matrix B – is also a column matrix that includes all the values at the right side of the equation. Example 1: 2 x 2 System Representation $\{x + y = 5\ 2x + 3y = 11$ Matrix Representation $Mat\ A = [1\ 1\ 2\ 3]$, $Mat\ X = [x\ y]$, $Mat\ B = [5\ 11]$ Example 2: 3 x 3 System Representation $\{x + y + z = 6\ 2x + 3y + z = 13\ x + 3y + 2z = 11$	**Activity B.1. EXPLORING KEY CONCEPTS** The goal of this section is pattern recognition. Learners often struggle with the transition because they view the x and y as essential anchors. By presenting these examples, you are teaching them that the position of a number in a matrix replaces the label of the variable. It is important to emphasize that you should first arrange the variables within the system properly. For the Matrix A, it includes all the coefficients of the system of linear equations. The first column includes the coefficient of the first variable, the second column includes all the coefficients of the second variable and so on. For Matrix X, it is a column matrix that includes all the variables involved within the system of linear equations. For Matrix B, it includes all the values at the right side of the equation.

	Matrix Representation $Mat\ A = [1\ 1\ 1\ 2\ 3\ 1\ 1\ 3\ 2]$, $Mat\ X = [x\ y\ z]$, $Mat\ B = [6\ 13\ 11]$ Example 3: 3 x 3 System Representation with Missing Variables $\{x + z = 4\ 2x + 3y + z = 13\ 3y + 2z = 7$ This can be rewritten as: $\{x + 0y + z = 4\ 2x + 3y + z = 13\ 0x + 3y + 2z = 7$ Matrix Representation $Mat\ A = [1\ 0\ 1\ 2\ 3\ 1\ 0\ 3\ 2]$, $Mat\ X = [x\ y\ z]$, $Mat\ B = [4\ 13\ 7]$	Explicitly point out what happens if a variable is missing (it becomes a 0) and that the "vertical bar" in the augmented matrix represents the equals sign.
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Matrix Row Operations** A row operation is an action applied to the rows of a matrix that transforms it without changing the solution set of the corresponding system of linear equations. ● Row Swapping ($R_i \leftrightarrow R_j$) – exchanging two rows of a matrix $[1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$ $R_1 \leftrightarrow R_2$ $[0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1]$ ● Row Multiplication (kR_i) – multiplying a non-zero scalar k to all the elements within the row $[1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$ $2R_1 \to R_1$ $[2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$ ● Row Addition ($kR_i + R_j \to R_j$) - Add a scalar multiple of one row to another row, and replace the later row with that sum $[1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$ $3R_1 + R_3 \to R_3$ $[1\ 0\ 0\ 0\ 1\ 0\ 3\ 0\ 1]$ **Elementary Matrix** An elementary matrix is a matrix formed by applying **exactly one** row operation on identity matrix.	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** Before diving into the operations, address the “why”. Learners often view these as arbitrary rules. Compare a matrix to a system of equations. Explain that swapping two equations or multiplying an entire equation by 2 doesn't change the solution. Emphasize that they aren't just changing numbers; they are "cleaning up" the matrix to reveal the answers. Row Swapping is essential for getting a non-zero number (ideally a 1) into the "pivot" position (the diagonal). Note that a common mistake is when learners often forget to swap the entire row. Row Multiplication is our tool for creating "1s." If a row starts with a 5, multiply by $1/5$ to get that leading 1. Remind LEARNERs that the scalar (k) cannot be zero. Ask them: "What happens to our information if we multiply an equation by 0?" (It disappears).

	Apply the following row operation to a 3 x 3 identity matrix to get the corresponding elementary matrix. 1. $3R_2 \rightarrow R_2$ 2. $R_1 \leftrightarrow R_3$ 3. $R_2 + R_3 \rightarrow R_3$ Identify the matrix row operation that resulted the following changes to the identity matrix. 1. $[0\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 0\ ]$ (Ans: $R_1 \leftrightarrow R_3$) 2. $[1\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ ]$ (Ans: $R_2 \leftrightarrow R_3$) 3. $[1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 5\ ]$ (Ans: $5R_3 \rightarrow R_3$) 4. $[-2\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ ]$ (Ans: $-2R_1 \rightarrow R_1$) 5. $[1\ 0\ 3\ 0\ 1\ 0\ 0\ 0\ 1\ ]$ (Ans: $3R_3 + R_1 \rightarrow R_1$) 6. $[1\ 0\ 0\ 0\ 1\ 0\ 0\ -2\ 1\ ]$ (Ans: $-2R_2 + R_3 \rightarrow R_3$) **Applying Sequence of Row Operations** Direction: Determine the sequence of row operations applied to the identity matrix (I_3) to obtain the following matrices. 1. $[1\ 3\ 5\ 0\ 1\ 0\ 0\ 0\ 1\ ]$ Answer: ● $3R_2 + R_1 \rightarrow R_1$ ● $5R_3 + R_1 \rightarrow R_1$ 2. $[1\ 0\ 3\ 0\ 2\ 0\ 0\ 0\ 1\ ]$ Answer: ● $2R_2 \rightarrow R_2$	Row Addition is considered as the powerhouse operation. It's how we create "0s" above and below our pivots. It is a combination of Scalar Multiplication and Addition. Note: Encourage learners to use scratch papers when dealing with matrices especially in operations. **Elementary Matrix** Start the lesson by challenging their understanding of I_n. Tell them that if I is the "identity," then an Elementary Matrix (E) is a "Matrix with a Memory." Direct them to look at each example on the worksheet and treat it like a "Spot the Difference" puzzle. Because they already know the 1s belong on the diagonal and 0s belong everywhere else, any deviation they see is a direct clue to the operation performed. Guide your learners to categorize the matrices based on the specific action done to the Identity Matrix. Use the following hierarchy: ● If the leading 1s have moved out of their diagonal home (as seen in Problems 1 and 2), direct them to identify this immediately as a Row Swapping. ● If the 1s are in the right place but one has changed into a different number (as seen in Problems 3 and 4), instruct them to label this as Row Multiplication. ● If a 0 has turned into a non-zero constant (as seen in Problems 5 and 6), they must recognize this as Row Addition. **Applying Sequence of Row Operations**

	• $3R_3 + R_1 \rightarrow R_1$ 3. $[0\ 0\ 1\ 0\ 3\ 0\ 4\ 0\ 0]$ Answer: • $R_1 \leftrightarrow R_3$ • $4R_3 \rightarrow R_3$ • $3R_2 \rightarrow R_2$ **Reduced Row Echelon Form (RREF)** A matrix is in reduced row echelon form if the entries along the main diagonal are all 1 and the rest of the entries are all 0. Sequence of row operations are applied to reduce a matrix to its reduced row echelon form. Example: Reduce the following matrices. 1. $[1\ 1\ 1\ -1]$ Solution: $[1\ 1\ 1\ -1] \quad -1R_1 + R_2 \rightarrow R_2 \quad [1\ 1\ 0\ -1]$ $[1\ 1\ 0\ -1] \quad R_2 + R_1 \rightarrow R_1 \quad [1\ 0\ 0\ -1]$ $[1\ 0\ 0\ -1] \quad -1R_2 \rightarrow R_2 \quad [1\ 0\ 0\ 1]$ 2. $[2\ 3\ 4\ 5]$ Solution: $[2\ 3\ 4\ 5] \quad -2R_1 + R_2 \rightarrow R_2 \quad [2\ 3\ 0\ -1]$ $[2\ 3\ 0\ -1] \quad 3R_2 + R_1 \rightarrow R_1 \quad [2\ 0\ 0\ -1]$ $[2\ 0\ 0\ -1] \quad \frac{1}{2}R_1 \rightarrow R_1 \quad [1\ 0\ 0\ -1]$	To effectively deliver this lesson on Sequences of Row Operations, you must transition your learners from seeing a matrix as a single "snapshot" of a rule to viewing it as the result of a mathematical narrative. Since your learners are already comfortable with individual elementary matrices, direct them to approach these complex matrices as "crime scenes" where multiple actions have occurred. Instruct them to perform a systematic Inventory of Deviations: have them scan the matrix row by row and compare it to the standard I_3. In Problem 1, for instance, they should notice that while the leading 1s are in their correct diagonal positions, Row 1 contains two clues, the numbers 3 and 5, which do not belong in the identity. Direct the learners toward the realization that each non-zero entry in a "zero-slot" represents a separate row addition operation that must be accounted for in the sequence. As you guide them through Problem 2 and Problem 3, emphasize the Hierarchy of Operations. Direct your learners to look for the most obvious changes first, such as a row swap or a scalar multiplication, before untangling the less obvious row additions. In Problem 3, for example, the primary change is the displacement of the 1s. Instruct the class to first identify the Row Swap $R_1 \leftrightarrow R_3$ as the foundational move. Once the structure of the matrix is moved, they can then identify the scalar multipliers (the 3 and the 4) that changed the original identity values. Finally, deliver the concept of Verification through Reconstruction. Direct the s to treat their proposed answers as a "recipe" that must be tested.

	$[1\ 0\ 0\ -1]\quad -1R_2 \to R_2\quad [1\ 0\ 0\ 1]$ 3. $[2\ 4\ 6\ 1\ 3\ 5\ 3\ 5\ 8]$ Solution: $[2\ 4\ 6\ 1\ 3\ 5\ 3\ 5\ 8]\quad R_2 \leftrightarrow R_1\quad [1\ 3\ 5\ 2\ 4\ 6\ 3\ 5\ 8]$ $[1\ 3\ 5\ 2\ 4\ 6\ 3\ 5\ 8]\quad -2R_1 + R_2 \to R_2\quad [1\ 3\ 5\ 0\ -2\ -4\ 3\ 5\ 8]$ $[1\ 3\ 5\ 0\ -2\ -4\ 3\ 5\ 8]\quad -3R_1 + R_3 \to R_3\quad [1\ 3\ 5\ 0\ -2\ -4\ 0\ -4\ -7]$ $[1\ 3\ 5\ 0\ -2\ -4\ 0\ -4\ -7]\quad -2R_2 + R_3 \to R_3\quad [1\ 3\ 5\ 0\ -2\ -4\ 0\ 0\ 1]$ $[1\ 3\ 5\ 0\ -2\ -4\ 0\ 0\ 1]\quad -\frac{1}{2}R_2 \to R_2\quad [1\ 3\ 5\ 0\ 1\ 2\ 0\ 0\ 1]$ $[1\ 3\ 5\ 0\ 1\ 2\ 0\ 0\ 1]\quad -3R_2 + R_1 \to R_1\quad [1\ 0\ -1\ 0\ 1\ 2\ 0\ 0\ 1]$ $[1\ 0\ -1\ 0\ 1\ 2\ 0\ 0\ 1]\quad R_3 + R_1 \to R_1\quad [1\ 0\ 0\ 0\ 1\ 2\ 0\ 0\ 1]$ $[1\ 0\ 0\ 0\ 1\ 2\ 0\ 0\ 1]\quad -2R_3 + R_2 \to R_2\quad [1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$ **Augmented Matrix** An augmented matrix is the matrix formed by appending the constant matrix B to the coefficient matrix A of a system of linear equations. It is written as $[A\|B]$. Thus, the augmented matrix combines the coefficient matrix A and the constant matrix B into a single matrix, which allows us to apply elementary row operations to solve the system.	After they derive a sequence, they should start with a fresh I_3 on their scratchpad and manually perform those steps. If their result does not match the worksheet, they have likely confused the the order of operations. **Reduced Row Echelon Form** You need to frame this not as a math problem, but as a strategic clean - up operation. Start by reminding your learners that while they have the three options of row operations, the real challenge is the sequence. Direct them to treat the matrix like a puzzle where the goal is to reveal the Identity Matrix hidden inside. You must be firm about the order of operations here. Tell them to work column by column, from left to right. If they try to fix a number in the third column before the first is finished, they'll end up in a loop situation where they keep breaking the zeros they just created. As you walk them through the worked examples from the slides, direct their attention to the "Pivot Strategy." Instruct them to always get their 1 first. Once they have a leading 1 in the top-left corner, emphasize that this 1 is now their tool to create zeros in the rest of that column. Note: Watch for learners who try to do mental math here. Insist them to write the "scratch row" off to the side. Key Delivery Checkpoints: ● Always secure a 1 in the diagonal before attempting to create 0s in that column. ● Once a column looks like the identity (1 on the diagonal, 0s elsewhere), it is locked. Tell them to never use a row operation that would change those established values.

	Example: Convert the following system of linear equation into a augmented matrix. 1. $\{x + y = 5\ 2x + 3y = 11$ Solution: Matrix Representation $Mat\ A = [1\ 1\ 2\ 3\ ]\ ,\quad Mat\ X = [x\ y\ ]\ ,\quad Mat\ B = [5\ 11\ ]$ Augmented Matrix $[B] = [\ 5\ 11\ ]$ 2. $\{x + y + z = 6\ 2x + 3y + z = 13\ x + 3y + 2z = 11$ Solution: Matrix Representation $Mat\ A = [1\ 1\ 1\ 2\ 3\ 1\ 1\ 3\ 2\ ]\ ,\quad Mat\ X = [x\ y\ z\ ]\ ,$ $Mat\ B = [6\ 13\ 11\ ]$ Augmented Matrix $[B] = [6\ 13\ 11\ ]$ **Solving Systems of Linear Equation through Matrices** Solving a system of linear equations using matrices and reduced row echelon form (RREF) is the process of representing the system as an augmented matrix and applying row operations to transform the matrix into reduced row echelon form, from which the values of the variables can be determined directly. Steps on solving linear equations using matrices:	**Augmented Matrix** Direct your leaarners to see the transition from a messy system of linear equations to a clean augmented matrix as a way to strip away the "noise" of variables like x, y, and z. You should explicitly instruct them that the vertical bar in $[A\|B]$ is not just a line, it is a mathematical "equal sign" that keeps the coefficients on the left and the constants on the right. **Solving Systems of Linear Equation through Matrices** You have to make sure your learners don't view the Augmented Matrix and RREF as two separate concepts. Direct them to see the Augmented Matrix as the "Before" picture and the Reduced Row Echelon Form as the "After" picture. Explain that by turning the left side of the bar into the Identity Matrix, we are essentially forcing the variables $x, y,$ and z to act as a dependent matrix and declare their values on the right side. Emphasize that by following the steps, the learners will have an additional guide on how to approach the problem.

Step 1. Identify Matrix A, Matrix X, and Matrix B based on $AX = B$.
Step 2. Form the augmented matrix $[A \mid B]$.
Step 3. Apply sequence of row operations (row swapping, scalar multiplication, and row addition) to reduced it to reduced row echelon form.
Step 4. The solution will be within the final matrix after it was reduced.

Example: Find the solution of the following systems of linear equation using reducing matrices into reduced row echelon form.

1. $\{x + y = 5\ 2x + 3y = 11$
Step 1. Identify Matrix A, Matrix X, and Matrix B based on $AX = B$.

$Mat\ A = [1\ 1\ 2\ 3\ ]\ ,\quad Mat\ X = [x\ y\ ]\ ,\quad Mat\ B = [5\ 11\ ]$
Step 2. Form the augmented matrix $[A \mid B]$.

$$[B] = [5\ 11\ ]$$

Step 3. Apply sequence of row operations (row swapping, scalar multiplication, and row addition) to reduced it to reduced row echelon form.

$$[5\ 11\ ] \quad -2R_1 + R_2 \rightarrow R_2 \quad [5\ 1\ ]$$
$$[5\ 1\ ] \quad -1R_2 + R_1 \rightarrow R_1 \quad [4\ 1\ ]$$

Step 4. The solution will be within the final matrix after it was reduced.

$$Reduced\ [B] = [4\ 1\ ]$$

$$x = 4 \quad , \quad y = 1$$

Since the first column is for the first variable (x) and the second column is for the second variable (y).

2. $\{x + y + z = 6\ 2x + 3y + z = 13\ x + 3y + 2z = 11$

Step 1. Identify Matrix A, Matrix X, and Matrix B based on $AX = B$.

$$Mat\ A = [1\ 1\ 1\ 2\ 3\ 1\ 1\ 3\ 2\ ]\ ,\qquad Mat\ X = [x\ y\ z\ ]\ ,$$
$$Mat\ B = [6\ 13\ 11\ ]$$

Step 2. Form the augmented matrix $[A \mid B]$.

$$[B] = [6\ 13\ 11\ ]$$

Step 3. Apply sequence of row operations (row swapping, scalar multiplication, and row addition) to reduced it to reduced row echelon form.

$$[6\ 13\ 11\ ] \quad -2R_1 + R_2 \rightarrow R_2 \quad [6\ 1\ 11\ ]$$

$$[6\ 1\ 11\ ] \quad -1R_1 + R_3 \rightarrow R_3 \quad [6\ 1\ 5\ ]$$

$$[6\ 1\ 5\ ] \quad -1R_2 + R_1 \rightarrow R_1 \quad [5\ 1\ 5\ ]$$

$$[5\ 1\ 5\ ] \quad -2R_2 + R_3 \rightarrow R_3 \quad [5\ 1\ 3\ ]$$

$$[5\ 1\ 3\ ] \quad \frac{1}{3}R_3 \rightarrow R_3 \quad [5\ 1\ 1\ ]$$

$$[5\ 1\ 1\ ] \quad R_3 + R_2 \rightarrow R_2 \quad [5\ 2\ 1\ ]$$

$$[5\ 2\ 1\ ] \quad -2R_3 + R_1 \rightarrow R_1 \quad [3\ 2\ 1\ ]$$

Step 4. The solution will be within the final matrix after it was reduced.

$$Reduced\ [B] = [3\ 2\ 1\ ]$$

$$x = 3\ ,\ y = 2\ ,\ z = 1$$

<table>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.

Direct your learners to view these systems not as algebra problems, but as structural puzzles that require a meticulous, step-by-step demolition. You must explicitly instruct them to begin by translating these systems into the Augmented Matrix format they've practiced. The teacher should circulate the room and ensure that every learner has correctly mapped the coefficients into their columns; if they miss a single variable in that initial setup, remind them that the entire solution they build afterward will be going towards the wrong way.

In problems 1 and 2, direct the learners to keep their eyes on the Identity Matrix target. Instruct them to narrate their row operations out loud or in the margins to prevent the mental fatigue that leads to sign errors.

In problems 3 and 4, you must manage their frustration levels. Direct the learners to treat the three-variable system as a marathon, not a sprint. Instruct them that the most common trap is trying to combine too many steps at once; tell them to perform exactly one row operation per new matrix drawing.

Finally, wrap up the session with a high-stakes Verification Check. Instruct every learner to take their final $x, y,$ and z values and plug them into the third equation of the original system.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTION

Instruction: Analyze each problem. Translate each situations into systems of linear equation and find its solution using matrices.
Jeepney, Tricycle, and Bus Fare
A commuter traveling to work in Manila used three types of transportation: jeepney, tricycle, and bus.
● On Monday, the commuter paid ₱115 for 2 jeepney rides, 1 tricycle ride, and 1 bus ride.
● On Tuesday, the commuter paid ₱150 for 1 jeepney ride, 2 tricycle rides, and 2 bus ride.
● On Wednesday, the commuter paid ₱115 for 3 jeepney ride, 1 tricycle ride, and 1 bus ride.

What is the fare for each type of transportation?
Rice, Corn, and Sugar Purchase
A customer bought goods from a market.</td><td>Activity C.1. MAKING REAL-WORLD CONNECTION

Start by emphasizing that the most critical phase of this activity is the translation from sentences to systems of linear equations. Direct the teacher to walk the learners through the "Jeepney, Tricycle, and Bus" scenario, instructing them to first explicitly define their variables, such as letting x represent the jeepney fare, y the tricycle fare, and z the bus fare.

As LEARNERs begin the heavy lifting of reducing these 3×3 matrices into Reduced Row Echelon Form (RREF), you must direct them to maintain extreme procedural discipline to avoid errors and loops in solution.

Instruct them to use the first row as their anchor to create zeros in the first column, specifically aiming to simplify the large totals.

Finally, direct the learners to interpret their final matrix not just as numbers, but as the actual prices they were seeking.</td></tr>
</table>

	• The customer paid ₱175 for 1 kg of rice, 2 kg of corn, and 1 kg of sugar. • Another customer paid ₱265 for 2 kg of rice, 1 kg of corn, and 2 kg of sugar. • A third customer paid ₱200 for 1 kg of rice, 1 kg of corn, and 2 kg of sugar. Find the price per kilogram of each product. **Filipino Street Food Sales** During a local event in a plaza, a vendor sold fish balls, kikiam, and squid balls. • One group paid ₱120 for 2 fish balls, 1 kikiam, and 3 squid balls. • Another group paid ₱125 for 1 fish ball, 3 kikiam, and 2 squid balls. • A third group paid ₱110 for 3 fish balls, 2 kikiam, and 1 squid balls. Determine the price of one order of each product.	**Processing Questions**: 1. When translating a real-world scenario into a system of equations, how do you determine which pieces of information become the coefficients in Matrix A and which become the constants in Matrix B? Describe how you ensure that each row and column in your augmented matrix represents a consistent category from the story. 2. After reaching the Reduced Row Echelon Form (RREF), how do you verify that the numerical values in the final column are logically sound within the context of the original problem? Why is it important to interpret these numbers back into their real-world labels (like price, weight, or distance) rather than just leaving them as variables $x, y,$ and z?
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Steps Summary:** 1. Represent system as augmented matrix. 2. Apply row operations to reach RREF. 3. Interpret solution from the RREF matrix. **Questions:** 1. Why is RREF more efficient than substitution or elimination for large systems? 2. How can this method save time in real-world applications like business, engineering, or scheduling? 3. What value does mastering row operations add to logical problem solving?	**Activity C.2. WRAPPING UP THE LESSON** Direct your learners to see the summary of steps, representing the system, applying operations, and interpreting the RREF, as a universal logical workflow for solving systems of linear equation using matrices. You should explicitly instruct them that the true value of mastering these matrices isn't just in the arithmetic, but in the efficiency, it provides over traditional substitution or elimination when dealing with the large, multi-variable datasets they will encounter in real - life. Direct the teacher to use the "Steps Summary" as a diagnostic checklist; if a learner is struggling, instruct them to pinpoint exactly which of the three stages is breaking down.
	C.3. Evaluating Learning	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS.	

	This activity must be completed individually by each learner. Its completion is essential for assessing their level of understanding of the lesson. Review their work promptly so you can provide timely and appropriate intervention for those who require additional support or remediation.
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. This exercise can be assigned as homework. Provide remediation for learners who have not yet mastered the lesson, and offer enrichment activities to those ready for further challenges..

III. CONTENT	**Lesson 3.4. MATRIX INVERSES AND LINEAR CLASSIFICATIONS**	
IV. OBJECTIVES	**At the end of this lesson the learners are expected to:** **1. define the inverse of a square matrix and state the identity property $AA^{-1} = I_n$.;** **2. calculate the inverse of matrices using the augmented matrix method $[A|I]$ through elementary row operations; and** **3. classify systems of linear equations as independent, dependent consistent, or inconsistent based on the results of row reduction and the inverse of the matrix.**	
V. PROCEDURES	**LEARNER ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1.*** **Eliciting *Prior Knowledge***	
	Activity A.1. **LEVELING LEARNERS READINESS: IDENTITY MATRIX QUEST** **Instruction:** Use the row operations (Row Swapping, Scalar Multiplication, and Row Addition) to reduce the following matrix to its corresponding identity matrix. 1. $\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}$ 2. $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\2 & 1 & 8\end{bmatrix}$ **Feedback and Reflection Mechanism** **Concept Check (Peer Feedback)** **Direction:** Check the box that best describes your experience: ☐ Smooth Sailing: I remembered the sequence (creating 1s then 0s) and applied operations without arithmetic errors.	**Activity A.1.** **LEVELING LEARNERS READINESS: IDENTITY MATRIX QUEST** This is a diagnostic phase. Walk around the room. If the learners cannot perform these row operations, the rest of the discussion will be challenging. Specifically, check if they are writing their operations. Explicitly tell them: "If you don't write the operations, it is prone to mistakes." You should walk around and observe LEARNER techniques, encouraging verbal explanation of each step. This activity bridges prior knowledge of RREF with inverses and system solutions. Solution: 1. $\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}$ $\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix} \quad -2R_1 + R_2 \to R_2 \quad \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$

	- Minor Detour: I performed the right operations but made a sign or multiplication error midway. - Roadblock: I struggled to decide which operation would create a zero without "breaking" the 1s I already created. **Processing Question:** How does the structure of the Identity Matrix serve as a 'map' for your row operations? In other words, how do you know when you are finished, and why is the order of your steps important?	$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix} \quad -3R_2+R_1\to R_1 \quad \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ 2. $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\2 & 1 & 8\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\2 & 1 & 8\end{bmatrix} \quad -2R_1+R_3\to R_3 \quad \begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\0 & 1 & 4\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\0 & 1 & 4\end{bmatrix} \quad -1R_2+R_3\to R_3 \quad \begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\0 & 0 & 1\end{bmatrix} \quad -3R_3+R_2\to R_2 \quad \begin{bmatrix}1 & 0 & 2\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 2\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad -2R_3+R_1\to R_1 \quad \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ To implement this activity on Feedback and Reflection Mechanism, begin by framing the Concept Check as a diagnostic tool to help you identify whether LEARNERs are struggling with basic arithmetic or the underlying logic of row reduction. After they complete the matrix problems, have them check the box that matches their experience and then pair up those who marked "Minor Detour" so they can act as "spotters" to find each other's calculation errors. For LEARNERs who hit a "Roadblock," provide immediate scaffolding by reminding them that a leading 1 must be established before clearing a column, which prevents them from "breaking" their previous work. Finally, use the "Processing Question" to facilitate a brief class discussion on how the Identity Matrix serves as a fixed destination that dictates their strategy. By asking them to explain why the order of steps is vital, you force them to move beyond rote calculation and into conceptual reasoning. Use the collective

		feedback from these reflections to determine if the class is truly ready to progress; if "Roadblocks" are common, perform a quick "think-aloud" demonstration on the board to clarify the procedural logic before starting your new lesson.
	A.2. Establishing the Purpose	
	Activity A.2: **APPRECIATING LESSON RELEVANCE: UNDOING THE OPERATION** Instruction: Solve the equation by finding the value of x: $3x = 12$ To find the value of x, divide both sides of equation by 3. $x = 4$ **Processing Questions:** 1. When solving the equation $3x = 12$, what operation did we perform to isolate x? 2. Why can we divide both sides by the same number? Now, let us consider this system of linear equations: $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ Matrix Representation of the system of equations: $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ Matrix Form: $AX = B$ **Think – Pair - Share:** 1. When equations are written in matrix form $AX = B$, what operation can help isolate X? 2. What do you think is the role of the inverse matrix in solving systems of linear equations? To isolate X, we multiply both sides by the inverse of the matrix A. $A^{-1}(AX) = B(A^{-1})$ $A^{-1}AX = A^{-1}B$ since $A^{-1}A = I$	**Activity A.2:** **APPRECIATING LESSON RELEVANCE: UNDOING THE OPERATION** Start by grounding your LEARNERs in the familiar world of basic algebra before you ever mention a matrix. Direct their attention to the simple equation $3x = 12$ and ask them to physically "undo" the multiplication. You want to emphasize that division is simply the inverse operation of multiplication; it's the tool we use to neutralize the 3 and isolate x. Once they've acknowledged that we divide both sides to maintain a balance, challenge them with a "what if" scenario: what if we had two variables and two equations? Show them the system of equations and then reveal the Matrix Representation. It is vital here that you explain that $AX = B$ isn't just a new way to write the problem, it's a way to treat an entire system of equations as if it were a single, simple algebraic sentence. Now, lead them into the "Think-Pair-Share" segment by drawing a direct parallel between the two images. Ask them, "If we used division to isolate x in the first problem, what is the 'matrix version' of division?" Guide them to see that since we can't "divide" matrices in the traditional sense, we use the Inverse Matrix A^{-1} instead. Explain that A^{-1} is the mathematical equivalent of an "undo" button.

	then $X = A^{-1}B$	
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. **EXPLORING KEY CONCEPTS: DISCOVERY THROUGH MATRIX MULTIPLICATION** **Instruction:** Multiply the following pair of matrices. 1. $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ 2. $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$ 3. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ 4. $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$ **Processing Questions:** 1. After multiplying Matrix A and Matrix B, what kind of matrix did you obtain? Describe the pattern of the numbers in the result. 2. What can you conclude about the relationship between Matrix A and Matrix B given that their product results in the identity matrix?	**Activity B.1.** **EXPLORING KEY CONCEPTS: DISCOVERY THROUGH MATRIX MULTIPLICATION** When you present these examples to your learners, your goal is to transition them from mechanical calculation to a discovery about the nature of inverse matrices. Don't give away the secret immediately; instead, let them work through the multiplication of the first pair. As they calculate the result for matrices A and B, you should circulate and watch for the moment they realize the product is the Identity Matrix. Use this as a pivot point to ask them what it feels like to multiply these specific numbers together. You want them to see that the "1" of the field of matrices is this specific diagonal pattern of ones and zeros. Connect this back to the broader goal of solving systems. You need to make sure they understand that finding this pair of matrices is the most critical step in isolating variables. Remind them that just as they used division in simple algebra to reach the number 1, they are using matrix multiplication here to reach the Identity Matrix. If they can grasp that $AB = I$ means B is the inverse of A, they are ready to proceed.
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Inverse of a Matrix** A square matrix A is said to have an inverse if there exists a square matrix B of the same order such that $$AB = BA = I$$ where I is the identity matrix of the same order.	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** **Inverse of a Matrix** Start by having your LEARNERs look at the expression $AB = BA = I$ and ask them to recall the commutative property they learned; point out that while matrix multiplication usually cares about order, the relationship between a matrix and its inverse is one of the rare instances where the order doesn't change the outcome.

	The matrix B is called the **inverse** of matrix A and is denoted by A^{-1}. If such a matrix does not exist, A is called **singular**; if it exists, A is called **invertible** or **non-singular**. Example: 1. The inverse of matrix $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ is the matrix $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$. $$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ 2. The inverse of matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is the matrix $A^{-1}=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. $$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **Finding the Inverse using the Augmented Matrix** Steps on finding the Inverse of a Matrix using the Augmented Matrix: 1. Write the augmented matrix $[A \mid I]$, where I is the identity matrix of the same order as A. 2. Use row operations to transform the left side of the augmented matrix into the identity matrix. 3. After the left side becomes I, the right side of the augmented matrix will be A^{-1}. If the left side cannot be transformed into I, the matrix is **non-invertible**. Example: Find the inverse of the following matrices using augmented matrix.	**Finding the Inverse using the Augmented matrix** Explain to your LEARNERs that by placing the original matrix A next to the Identity matrix I of the same order, they are creating a system where every operation applied to the left side to reach reduced row-echelon form must be mirrored on the right side. Stress that the goal is a total transformation: once the left side is converted into I through valid row operations, the right side is mathematically guaranteed to be A^{-1}. When your LEARNERs reach the end of the calculation, teach them the process of checking their answers. They should multiply their resulting A^{-1} by the original A. If the product is not the Identity matrix, they have a calculation error. Point out that the

	1. $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ Step 1: Write the augmented matrix $[A \mid I]$, where I is the identity matrix of the same order as A. $$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$$ Step 2: Use row operations to transform the left side of the augmented matrix into the identity matrix. $$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right] \quad \frac{1}{2}R_1 \to R_1 \quad \left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$$ $$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 5 & 3 & 0 & 1 \end{array}\right] \quad -5R_1 + R_2 \to R_2 \quad \left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right]$$ $$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{5}{2} & 1 \end{array}\right] \quad 2R_2 \to R_2 \quad \left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -5 & 2 \end{array}\right]$$ $$\left[\begin{array}{cc|cc} 1 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -5 & 2 \end{array}\right] \quad -\frac{1}{2}R_2 + R_1 \to R_1 \quad \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$ Step 3: After the left side becomes I, the right side of the augmented matrix will be A^{-1}. $$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right] \quad \to \quad A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$ 2. $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Step 1: Write the augmented matrix $[A \mid I]$, where I is the identity matrix of the same order as A. $$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$	most common mistakes occur in the right-hand side of the augmented matrix because LEARNERs often lose focus on the "I" side while concentrating on the "A" side. Remind them that the horizontal bar or dashed line is just a visual separator.

Step 2: Use row operations to transform the left side of the augmented matrix into the identity matrix.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad -2R_2 + R_1 \to R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad R_3 + R_1 \to R_1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad -1R_3 + R_2 \to R_2 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Step 3: After the left side becomes I, the right side of the augmented matrix will be A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \to \quad A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Checking for Classification of Systems of Linear Equations using Matrix Inverses

1. If the inverse exist, then the system is and **Independent System (Unique Solution)**
2. If the inverse does not exist, then the system of linear equation is either **Dependent Consistent System (Infinitely Many Solutions)** or an **Inconsistent System (No Solution)**.

If the inverse does not exist, use the row operations to determine whether it is Dependent Consistent system or Inconsistent System.

Example:

Independent System (Unique Solution)

Checking for Classification of Systems of Linear Equations using Matrix Inverses

When you deliver this lesson on classifying systems of linear equations via matrix inverses, you must anchor the entire discussion in the binary outcome of the reduction process. Instruct your LEARNERs that the existence of an inverse is the primary diagnostic tool for determining the nature of a system. Explicitly state that if the coefficient matrix A can be reduced to the identity matrix I, then the system is Independent and possesses a single, unique solution. Direct their attention to the final row-echelon form; if the identity appears on the left, the values on the right are the only possible coordinates for the solution.

	$\begin{cases} x + y + z = 6 \\ 2x - 3y + z = 3 \\ x + 2y - 2z = 3 \end{cases}$ Augmented Matrix: $\left[\begin{array}{ccc\|c} 1 & 1 & 1 & 6 \\ 2 & -3 & 1 & 3 \\ 1 & 2 & -2 & 3 \end{array}\right]$ Reduced Augmented Matrix: $\left[\begin{array}{ccc\|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$ Since the coefficient matrix within the augmented matrix is reduced to its identity, therefore the system is Independent or has a unique solution. **Dependent Consistent System (Infinitely Many Solutions)** $\begin{cases} x + y + z = 3 \\ 2x + 2y + 2z = 6 \\ x - y + z = 1 \end{cases}$ Augmented Matrix: $\left[\begin{array}{ccc\|c} 1 & 1 & 1 & 3 \\ 2 & 2 & 2 & 6 \\ 1 & -1 & 1 & 1 \end{array}\right]$ Reduced Augmented Matrix: $\left[\begin{array}{ccc\|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ Since there is a row that has all zero entries ($0 = 0$), then the system of linear equation is dependent consistent or has infinitely many solutions. **Inconsistent System (No Solution)** $\begin{cases} x + y + z = 4 \\ 2x + y + z = 6 \\ x + 2y + z = 7 \end{cases}$ Augmented Matrix:	Transition immediately to the scenario where the inverse does not exist, which occurs when a row of zeros appears on the left side during reduction. You must train your LEARNERs to look past the left-hand side and analyze the resulting constant on the right-hand side of that specific row. Explain that this is where the classification splits into two distinct paths. If the reduction results in a row of all zeros across both sides ($0 = 0$), the system is Dependent Consistent, meaning the equations are redundant and there are infinitely many solutions. But, if the reduction produces a row of zeros on the left but a non-zero constant on the right ($0 = c, where\ c\ is\ non - zero$), the system is Inconsistent, meaning the planes or lines never intersect and no solution exists.

	$\left[\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 6 \\ 1 & 2 & 1 & 7 \end{array}\right]$ Reduced Augmented Matrix: $\left[\begin{array}{ccc\|c} 1 & 1 & 1 & 4 \\ 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right]$ Since there is a row that has all zero entries on the left and mapped to a non – zero value on the right ($0 = C, where\ c\ is\ non - zero$), then the system of linear equation is an inconsistent system or has no solutions.	
	B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)	
	Activity B.3. **PRACTICING LEARNED SKILLS: INVESTIGATIVE TABLE INVOLVING MATRICES** Before facilitating this activity, ensure that all learners have a **copy** of the LAS. Your primary objective is to direct their attention to the structural relationship between matrix invertibility and system consistency. Instruct them to begin by representing each system as an augmented matrix $[A\|I]$. You must emphasize that their first goal is to determine the existence of an inverse by attempting to transform the left side into the Identity matrix. If they achieve an identity matrix, instruct them to identify the system as Independent and extract the unique solution from the right-hand column. However, if they encounter a row of zeros on the left, tell them they must analyze the corresponding value on the right. If that value is zero, they should classify the system as Dependent Consistent because the statement implies infinite solutions. If the value is non-zero, they must classify the system as Inconsistent because a statement like 0 = 8 is a contradiction, proving no solution exists.	
C. Demonstrating Knowledge and Skills	***C.1. Finding Practical Application***	
	Activity C.1. **MAKING REAL-WORLD CONNECTION: FUNDRAISER SUPPLY AUDIT** **Instructions**: Read the scenario and perform the indicated tasks. Scenario: You are ordering T-shirts x, Caps y, and Lanyards z for three different school clubs. You must check the order logs to see if the quantities are solvable. The logs are represented with the following systems of linear equation. Task:	**Activity C.1.** **MAKING REAL-WORLD CONNECTION: FUNDRAISER SUPPLY AUDIT** Direct your LEARNERs to act as Logistics Managers for a school fundraiser. Tell them that the augmented matrix is a diagnostic tool used to check if a supply request is valid. Instruct them to look at the left side of the matrix first: if it can become the identity matrix, the inverse exists, meaning the order is clear and has exactly one solution. If they find a row of zeros on the left, the matrix is non-invertible, and they must use the constant on the right to decide if the order is redundant or impossible. Your

	For each order log, convert the system to an augmented matrix, perform the row operation, determine whether its inverse exists and explain what the result means for the club. Here are your order logs: Order Log 1 $\begin{cases} x + y + z = 30 \\ y + 2z = 20 \\ 2z = 10 \end{cases}$ Order Log 2 $\begin{cases} x + y + z = 30 \\ y + z = 15 \\ 2y + 2z = 30 \end{cases}$ Order Log 3 $\begin{cases} x + y + z = 30 \\ y + z = 15 \\ 2y + 2z = 50 \end{cases}$ Processing Questions: 1. While solving these problems, if you had encountered a row of all zeros on the left side of the augmented matrix (the side that should become the identity), what would that tell you about the existence of the inverse? How does a row of zeroes impact your ability to classify the system of equations? 2. After completing the row operations to find A^{-1} for the 3x3 matrix in Problem #3, what is the most reliable way to verify your answer is correct using the identity property? Why is it important to perform this check before using the inverse to solve a system?	delivery should be direct: "The left side tells you if the math works; the right side tells you if the situation is possible in the real world." In Order Log 1, The order is Perfect. There is exactly one specific number of shirts, caps, and lanyards that fits this record. You can proceed with the purchase. In Order Log 2, The order is Flexible. Log 2 and Log 3 are identical, meaning you don't have enough unique information to find one single answer. There are infinite combinations of items that could work. In Order Log 3, The order is Impossible. The records contradict each other. You must cancel the order and fix the data.
	C.2. Making Generalization	
	Activity C.2. **WRAPPING-UP THE LESSON: MATRIX DIAGNOSTIC SUMMARY** **Processing Questions:** • **General Rule of Invertibility** 1. What does a diagonal of 1's indicate regarding the invertibility of matrices?	**Activity C.2.** **WRAPPING-UP THE LESSON: MATRIX DIAGNOSTIC SUMMARY** To deliver this activity effectively, you must focus the learners' attention on the structural relationship between a matrix's invertibility and the logic of the resulting system.

2. What if a row of zeroes appear? Is it still possible that a matrix is still invertible?

Based on the activities, we can assume that:

1. The Matrix is Invertible (Inverse exists) if we can successfully create a diagonal of 1's on the left side.
2. The Matrix is Non-Invertible (No inverse) if any row on the left side reduces to all zeros.

- **Classification of Matrices based on its Inverse**
  1. Can you classify a system of linear equations: Independent, Dependent Consistent, and Inconsistent based on its inverse? How and what are the constraints to consider?

Resulting Row	Inverse Status	System Classification	Logic Check
Diagonal of 1's	Exists	Independent	Valid
A row of zero appears and is mapped to zero	No Inverse	Dependent	0 = 0 (Always True)
A row of zero appears and mapped to non – zero scalar	No Inverse	Inconsistent	0 = c (Impossible)

Processing Questions:

1. If you find a row of zeros on the left side, do you even need to try and find the inverse anymore?
2. Can a system be Inconsistent (No Solution) if the inverse exists?
3. Why does $0 = c$ mean 'No Solution' while $0 = 0$ means 'Infinite Solutions'?

Instruct your LEARNERs to begin by translating each system into an augmented matrix and immediately identify the diagonal elements. Direct them to perform row operations with the singular goal of reaching the identity matrix on the left side. You should emphasize that the moment a row of zeros appears on the left side of the vertical bar, the matrix is technically non-invertible, and they must stop searching for an inverse.

At this juncture, direct them to shift their analysis to the constant on the right side of that zeroed row to determine the system's consistency.

Explain to the class that if the left-side zero row corresponds to a zero on the right, they have encountered a "true" identity statement, which characterizes a Dependent Consistent system with infinite solutions. Conversely, if the zero row corresponds to a non-zero constant, they must recognize this as a mathematical contradiction, which identifies the system as Inconsistent with no possible solution.

You must ensure that they do not equate "No Inverse" exclusively with "No Solution"; rather, they must use the constant as the definitive tie-breaker. By enforcing this binary check, first for invertibility on the left, then for logic on the right, you provide them with a concrete, repeatable method for system classification that relies on mathematical evidence rather than intuition.

C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)

Activity C.3
ASSESSING LEARNING OUTCOMES: SYSTEM DIAGNOSTIC AUDIT

Before facilitating this activity, ensure that all learners have a **copy** of the LAS.

	To evaluate learner mastery, you must direct them to perform a full system audit on three distinct cases. Instruct your LEARNERs that they are not merely solving for $x, y,$ and z, but are diagnosing the mathematical structure of each system. Direct them to prioritize the "Left Side" of the augmented matrix to determine invertibility. You must emphasize that a row of zeros on the left side is the definitive evidence of a non-invertible matrix. Once they identify a non-invertible state, direct them to use the "Right Side" constant to decide if the system is logically consistent (Dependent) or logically impossible (Inconsistent). Ensure they articulate the "Why" for each classification using the logic checks to prove they understand the underlying principles of the lesson. You can ask the processing questions.
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)
	Activity C.4. **EXTENDING AND REINFORCING LEARNING: MATRIX TRUTH DETECTOR** Before facilitating this activity, ensure that all learners have a copy of the LAS. When addressing struggling learners, you must strip away the complexity of 3x3 calculations and focus entirely on visual pattern recognition. Direct your learners to look at the "Bottom Row" of the matrix as a "Truth Detector." Instruct them that they do not need to solve the entire matrix to understand the system's nature; they only need to interpret the final row. Use a face-to-face, direct approach: "Ignore the top two rows for a moment. Look at the last row. Does it make sense, is it a repeat, or is it a lie?" Your goal is to build their confidence in identifying Invertibility and Classification by looking at the relationship between the zero and the constant.

III. CONTENT	**Lesson 3.5. DETERMINANT OF A 2X2 MATRIX**	
IV. OBJECTIVES	**By the end of the discussion, learners should be able to:** **1. define the determinant of a 2×2 matrix;** **2. compute the determinant correctly; and** **3. apply the concept of determinants in real-world problems.**	
V. PROCEDURES	**ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS **Instructions:** 1. For each system of linear equations given: 2. Rewrite the system as an augmented matrix. 3. Use elementary row operations (row switching, row multiplication, or row replacement) to simplify the matrix 4. Continue applying row operations until the matrix is a simplified form. 5. Examine the final matrix and classify the system as:	**Activity A.1. LEVELING LEARNER READINESS** 1. Prepare the Learners a. Recall briefly that systems of linear equations can be written as augmented matrices. b. Review the three elementary row operations: row switching, row multiplication by a nonzero constant and row replacement c. Emphasize that these operations do not change the solution set of the system.

	a. Independent if has a unique solution, b. Dependent consistent if has infinitely many solutions, or c. Inconsistent if has no solution. Given: 1. $x+y=3$ $x-y=1$ 2. $x+2y=4$ $2x+4y=8$ 3. $2x-y=3$ $4x-2y=5$	2. Present the Activity a. Display or distribute the given systems of linear equations. b. Instruct learners to work individually first, then allow them to compare answers with a partner for checking. 3. Remind learners to: a. Write the augmented matrix correctly before applying row operations. b. Perform row operations step by step and show their work. 4. Circulate around the room to: a. Check correct formation of augmented matrices. b. Address common errors, such as incorrect row replacement or arithmetic mistakes. 5. Facilitate Analysis and Classification: Ask probing questions while learners work, such as: a. "What does the appearance of all zeros in a row tell you?" b. "What does a row like [0 0 | c], where c ≠ 0, indicate?" c. Encourage learners to focus on the final row reduced form, not just the calculations. 6. Discuss the Results a. After most learners have finished, conduct a class discussion: b. Let selected learners present their reduced matrices. c. Ask them to explain how the final form led them to classify the system. d. Reinforce the correct classifications: • Independent system → unique solution (pivot in each variable column) • Dependent consistent system → infinitely many solutions (free variable) • Inconsistent system → no solution (contradictory row) 7. Provide Feedback and Clarification a. Affirm correct reasoning and correct misconceptions immediately. b. Highlight that row operations allow learners to determine solution behavior without graphing. 8. Bridge to the Next Lesson
	A.2. Establishing the Purpose of the Lesson	

<table>
<tr><td></td><td>Activity A.2. APPRECIATING LESSON RELEVANCE

Instructions:
1. Observe the 2×2 matrices written on the board.
$\begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$
2. Count the number of elements in each matrix.
3. Answer the teacher's questions regarding whether the matrix can be represented by one value.
4. Listen carefully as the teacher introduces the term determinant.</td><td>Activity A.2. APPRECIATING LESSON RELEVANCE

1. Write the following 2×2 matrix on the board:
$\begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}$
2. Ask the learners the following questions:
a. "How many elements does this matrix have?
b. "Can this matrix be represented by a single number?"
3. Present another 2×2 matrix:
$\begin{bmatrix}5 & 2\\1 & 6\end{bmatrix}$
4. Ask:"If we are given a specific rule, do you think we can compute one value from any 2×2 matrix?"
5. Introduce the term determinant as the name of this special value without yet explaining the formula.
6. Clearly state the purpose of the lesson:</td></tr>
<tr><td>B. Instituting New Knowledge</td><td colspan="2">B.1. Presenting Examples</td></tr>
<tr><td></td><td>Activity B.1. EXPLORING KEY CONCEPT

Computing the Determinant of a 2x2 Matrix

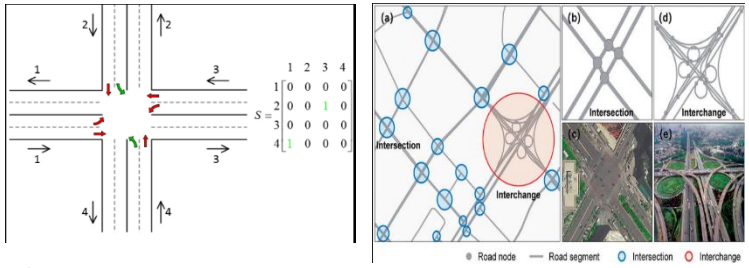
Instructions:
1. Observe the general form of a 2×2 matrix written on the board.
2. Take note of the formula for the determinant: $ad - bc$.
3. Follow the teacher as each example is solved step by step.
4. Identify:
a. the elements of the main diagonal,
b. the elements of the other diagonal.
5. Compute the determinant by multiplying and subtracting as shown.
6. Write the final answer clearly and label it as the determinant.
7. Answer the guide questions posed by the teacher
8. Consider a 2×2 matrix:
$A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

The determinant of matrix A, denoted as $\det(A)$or $\mid A \mid$, is calculated as:</td><td>Activity B.1. EXPLORING KEY CONCEPT

1. Present the general form of a 2×2 matrix on the board:
$A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$
2. Introduce the determinant of a 2×2 matrix and write its notation and formula:
$\det(A) = ad - bc$
3. Provide the first example:
$1.\ A = \begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}$
Solve step by step:
a. Multiply the elements of the main diagonal: (2)(1) = 2
b. Multiply the elements of the other diagonal: (3)(4) = 12
c. Subtract: $2 - 12 = -10$
4. Emphasize that the resulting value is called the determinant of the matrix.</td></tr>
</table>

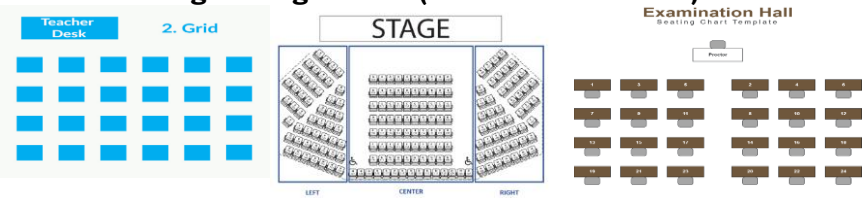
	$\det(A) = ad - bc$ *Example: Calculate,* 1. $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ 2. $B = \begin{bmatrix} 5 & 2 \\ 1 & 6 \end{bmatrix}$	5. Present a second example: 2. $B = \begin{bmatrix} 5 & 2 \\ 1 & 6 \end{bmatrix}$ 6. Ask guiding questions while solving: a. "What values are multiplied first?" b. "Which operation do we perform last?" 7. Reinforce the idea that the same procedure applies to **any** 2×2 matrix.
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Instructions:** 1. Work individually or with a partner. 2. You will provide the following scenario, find determinant of each 2×2 matrix using the formula $det\ (A) = ad - bc$D **Scenarios** a. Two rows of the matrix are exactly the same. b. One row is multiplied by a constant. c. Two rows are swapped. d. One row contains all zeros. e. One row is a multiple of the other. f. Columns are switched. 3. Answer the following questions: a. When does the determinant becomes positive? b. When does the determinant becomes negative? c. When does the determinant becomes zero? **2x2 Matrix and Its Determinants and Properties** **Interpreting the Determinant** **a. If the determinant is positive** ● The matrix preserves orientation. ● The transformation does not "flip" the shape. **b. If the determinant is negative** ● The matrix reverses orientation (like a reflection or flip).	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** 1. Ask the learners to work individually first, then compare answers with their group to work on the different scenarios provided 2. Facilitate the activity Walk around as learners work and guide them by asking questions such as: a. "What happens to the determinant if you repeat a row?" b. "What did you notice about determinants when rows were swapped earlier?" c. "Does multiplying a row change the entire determinant the same way?" This encourages deeper conceptual reasoning. 3. Conduct Class Checking and Discussion Go through the answers one by one: a. Ask learners which property they matched with each scenario. b. Invite volunteers to explain their answers. c. Clarify misconceptions and reinforce the correct **Discussion on 2x2 Matrix and Its Determinants and Properties** 1. Connect the Discussion to Learner's Prior Activity Begin by recalling the patterns leaqrners discovered during the Discovery Activity. Ask questions such as: a. What did you observe when two rows were identical? b. What happened when row were swapped?

	• The shape is turned "inside out." **c. If the determinant is zero** • The rows or columns are **dependent** (one is a multiple of the other). • The matrix is **not invertible**. • The transformation collapses the shape into a line or a point. **Important Properties of 2×2 Determinants** These properties connect directly to the patterns discovered in the earlier activity. **a. Identical rows or columns → determinant = 0** Because there is no unique direction (rows/columns overlap), the area collapses. **b. Proportional rows or columns → determinant = 0** Example: [4]and [8] One row is double the other → dependent. **c. Swapping two rows or columns flips the sign** $det\ (A_{swapped}) = -det\ (A)$ **d. Multiplying a row by a constant multiplies the determinant by that constant** $det\ (kR_1) = k \cdot det\ (A)$ **e. A diagonal 2×2 matrix has determinant equal to the product of diagonal entries** $[a\ 0\ 0\ d\ ] \rightarrow det\ = ad$ **Example**. Compute the determinant of each matriux 1. $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ 2. $B = \begin{bmatrix} 7 & -1 \\ 0 & 6 \end{bmatrix}$ 3. $C = \begin{bmatrix} -3 & 2 \\ 5 & -1 \end{bmatrix}$ 4. $D = \begin{bmatrix} 0 & 8 \\ -4 & 3 \end{bmatrix}$ 5.	c. When did the determinant become zero? 2. Present the definition of a determinant. Guide the learners conceptually. 3. Discuss Key Properties using LEARNER observations. Revisit the patterns from the earlier activity and link them to the formal rules. 4. Transition to Activity 2. Tell Learners that they will now apply the formal rules and understanding in computations and concepts analysis **Solution to the examples.** **1.** $det\ (A) = 3(4) - 5(2) = 12 - 10 = 2$ 2. $det\ (B) = 7(6) - (-1)(0) = 42 - 0 = 42$ 3. $det\ (C) = (-3)(-1) - 2(5) = 3 - 10 = -7$ 4. $det\ (D) = 0(3) - 8(-4) = 32$ 32 5. $det\ (E) = 6(5) - 0(-2) = 30$ 30 6. $det\ (F) = (-1)(-2) - 4(3) = 2 - 12 = -10$ 7. $det\ (G) = 9(1) - 2(-3) = 9 + 6 = 15$ 8. $det\ (H) = 4(2) - (-5)(7) = 8 + 35 = 43$ • .

	6. $E = \begin{bmatrix} 6 & 0 \\ -2 & 5 \end{bmatrix}$ 7. $F = \begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix}$ 8. $G = \begin{bmatrix} 9 & 2 \\ -3 & 1 \end{bmatrix}$ $H = \begin{bmatrix} 4 & -5 \\ 7 & 2 \end{bmatrix}$	
	B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)	
	Activity B.3. PRACTICING LEARNED SKILLS Before facilitating this activity, ensure that all learners have a copy of the LAS. Guide learners at the start, then allows learners to work independently while providing quick feedback. This structured drill strengthens procedural fluency, preparing LEARNERs for upcoming lessons on matrix properties and applications.	
C. Demonstrating Knowledge and Skills	***C.1. Finding Practical Application***	
	Activity C.1. MAKING REAL-WORLD CONNECTION **Visual Examples of Real-Life Applications of Matrices and Determinants** **1. Traffic Mapping (Road Networks)**  **Explanation:** Transportation networks can be represented using **time–space matrices**, where rows and columns represent time and road segments while matrix values represent traffic variables. Reference / Source:	**Activity C.1. MAKING REAL-WORLD CONNECTION** **Visual Examples of Real-Life Applications of Matrices and Determinants** **1. Traffic Mapping (Road Networks** **Explanation for Learners:** Traffic systems in cities can be represented using **matrices**. Each intersection and road connection can be organized in rows and columns. Traffic engineers use matrices to analyze traffic flow, optimize routes, and reduce congestion. **Example teacher statement:** "Each intersection can be represented as a point, and the connections between them can be organized in a matrix to study traffic flow." **2. Seating Arrangements (Rows and Columns)** **Explanation for learners:** Seating arrangements in classrooms, theaters, or exam halls

Ma, X., Dai, Z., He, Z., Na, J., Wang, Y., & Wang, Y. (2017). *Learning Traffic as Images: A Deep Convolutional Neural Network for Large-Scale Transportation Network Speed Prediction.* Sensors.

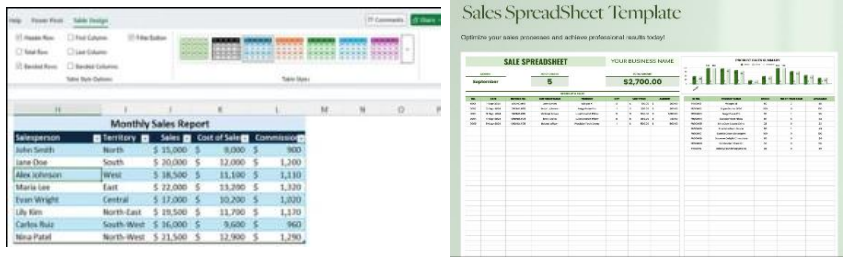
2. Seating Arrangements (Rows and Columns)



Explanation:
Seating charts visually represent the arrangement of chairs or desks in rows and columns, similar to the structure of a matrix.
Additional explanation:
A classroom seating layout can also be treated like a two-dimensional array or matrix, where each seat is identified by its row and column position
Reference / Source:

- EdrawSoft. *Seating Chart Examples and Templates.*

3. Business Data



Explanation:
Digital images are represented as matrices where each element corresponds to a pixel value, which stores color or intensity information
Reference / Source:
Math Warehouse. *Matrix Definition and Examples.*

are naturally arranged in rows and columns, which form a matrix structure.

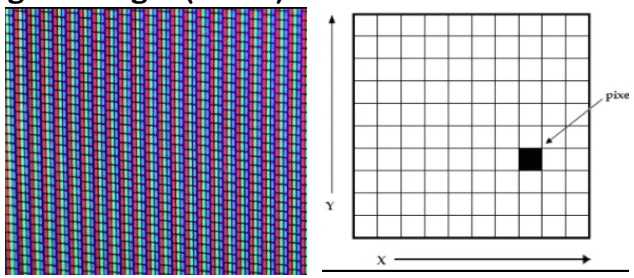
Example teacher statement:
"If we label rows and columns, every seat has a position like (row, column). That is exactly how matrices are organized."

3. Business Data Tables
Explanation for Learners:
Companies organize information like sales, expenses, and inventory in tables that look like matrices. Each row and column contains specific data used for analysis and decision-making.

Example teacher statement:
"Spreadsheets in business work very much like matrices because they store data in rows and columns."

4. Digital Images (Pixels)
Explanation for Learners:
Digital images are made of tiny squares called pixels arranged in rows and columns. Each pixel stores color values, forming a matrix of numbers that computers process to display images.

Example teacher statement:
"When you zoom into a digital image, you will see tiny squares. These squares form a matrix that computers use to display pictures."

	4. Digital Images (Pixels)  **Explanation:** Digital images are represented as matrices where each element corresponds to a pixel value, which stores color or intensity information Reference / Source: • Pesco, D. U., & Bortolossi, H. J. *Matrices and Digital Images.* Klein Project Blog. • Digital Image Fundamentals (Computer Vision lecture materials).	
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Instructions**: 1. Examine the following situations and explain what each tells you about the transformation: • The determinant is positive. • The determinant is negative. • The determinant is zero. 2. Analyze these cases and identify the underlying principle that connects them. 3. Formulate a general statement that describes how the determinant reflects orientation, dependence, and invertibility of a matrix. 4. Discuss with your group: • How do identical or proportional rows/columns affect the determinant? • Why does swapping rows or columns change the sign but not the magnitude?	**Activity C.2. WRAPPING UP THE LESSON** 1. Guide learners to reason beyond computation — focus on why the determinant behaves this way. 2. Ask probing questions such as: • "How does the determinant act as a test for independence?" • "What geometric meaning does a sign change represent?" • "Can you predict what happens to the shape when the determinant equals zero?" 3. Encourage learners to express their insights using their own reasoning and expressions

	• What does this imply about the geometry of the transformations.	
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. **ASSESSING LEARNING OUTCOMES: DETERMINANT CHECKPOINT** Before facilitating this activity, ensure that all learners have a **copy** of the LAS. 1. Use this activity to check if learners understand both how to compute determinants and what they mean. 2. Encourage learners to explain their reasoning briefly but clearly. 3. Allow 10–15 minutes to complete the activity. 4. Review the answers quickly and clarify misconceptions before ending the lesson**.**	
	C.4. Additional Activities	
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a **copy** of the LAS. 1. Emphasize that this activity aims to reinforce and apply concepts, helping learners recognize patterns and interpret determinant values in different contexts. 2. Guide learners to reason beyond computation, focusing on how determinant values affect transformation, orientation, and invertibility. 3. Ask analytical questions such as: a. "What does a positive determinant tell us about the transformation?" b. "How does a zero determinant relate to dependence between rows or columns?" c. "Why does swapping rows or columns change the sign but not the magnitude?"	
III. CONTENT	**Lesson 3.6. DETERMINANT OF A 3×3 MATRIX**	
IV. OBJECTIVES	**At the end of the lesson, learners should be able to:** **1. compute determinants of 3×3 matrices using Sarrus' Rule:** **2. apply the cofactor (minor) expansion along any row or column to find the determinant systematically; and** **3. solve real-world problems involving 3x3 matrices.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. **LEVELING LEARNERS READINESS: MATRIX MATCH-UP**	**Activity A.1.** **LEVELING LEARNERS READINESS: MATRIX MATCH-UP**

	Instructions: 1. Form groups of three. Each group receives five matrix cards. 2. Recall and discuss: Within your group, review how to find the determinant of a 2×2 matrix using the formula *det(A)=ad−bc*. 3. For each matrix card, compute its determinant. **Matrix card** $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 3 \\ 2 & 5 \end{bmatrix}$ $D = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$ $E = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}$	1. Prepare the materials a. Whiteboard or projector b. Marker pens c. Prepared matrix cards (each card shows a 2×2 matrix) 2. Encourage learners to verbalize their reasoning rather than just compute. 3. Ask probing questions like: a. "What does a determinant of zero tell us about the matrix?" b. "How might this relate to geometric interpretation?" 4. Use this discussion to transition smoothly into the next topic—determinant as a measure of area or transformation.
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. **APPRECIATING LESSON RELEVANCE: WHAT DOES A 3×3 DETERMINANT REPRESENT?** **Instructions**: 1. Think back to what you learned about 2×2 determinants and area. 2. Discuss with your group: • "If area changes in 2D, what changes in 3D?" • "How might a 3×3 matrix describe that change?" 3. Write your group's answer on the board or share aloud. 4. Listen as your teacher explains how determinants extend from 2D to 3D	**Activity A.2.** **APPRECIATING LESSON RELEVANCE: WHAT DOES A 3×3 DETERMINANT REPRESENT?** 1. Recall prior concept: Ask: "If a 2×2 determinant tells us how *area* is scaled, what do you think a 3×3 determinant tells us?" *Annotation:* Encourage learners to reason from geometric intuition rather than formulas. 2. Guide discussion: a. Let learners share ideas (e.g., "space," "volume," "3D shape"). b. Confirm that the determinant of a 3×3 matrix represents *volume scaling* in three-dimensional space. 3. Connect to lesson goal: Say: "Today, we'll extend determinants to 3×3 matrices using two methods—expansion and shortcut rules. These help us find how transformations affect volume." 4. Visual reinforcement (optional): Show a cube or rectangular prism and explain that changing its dimensions can be modeled by a 3×3 matrix—its determinant tells how the volume changes.
	B.1. Presenting Examples	

B. Instituting New Knowledge

Activity B.1.
EXPLORING KEY CONCEPTS: UNLOCKING THE 3×3 MATRIX

Instructions:

1. With your group, observe the the 3×3 matrix:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

2. Label each element with its position (e.g., a11, a12, a13). a_{11}, a_{12}, a_{13}).
3. Match each vocabulary term with its meaning in your notebook.

Term	Meaning
Order of a matrix	a. A rectangular array of numbers arranged in rows and columns.
Element / Entry	b. Each number inside the matrix.
Column	c. A horizontal arrangement of elements.
Matrix	d. A vertical arrangement of elements.
Row	e. The number of rows and columns (e.g., 3×3 means 3 rows and 3 columns).

4. Discuss the following with through these processing questions.
   a. How many elements are there in a 3×3 matrix?
   b. How do we identify the position of each element?
   c. What makes a 3×3 matrix different from a 2×2 matrix?
   d. Where might we encounter 3×3 matrices in real life.

Activity B.1.
EXPLORING KEY CONCEPTS: UNLOCKING THE 3×3 MATRIX

3. Introduce the concept visually: Display a 3×3 matrix on the board:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

4. Explain that each element a_{ij} represents a value located at row i and column j. This helps learners see the systematic arrangement of data.
5. Introduce and define key terms to reinforce mathematical language for later use
   a. Matrix: A rectangular array of numbers arranged in rows and columns.
   b. Element/Entry: Each number inside the matrix.
   c. Row: A horizontal arrangement of elements.
   d. Column: A vertical arrangement of elements.
   e. Order of a Matrix: The number of rows and columns (e.g., 3×3 means 3 rows and 3 columns).
6. Guide observation by asking the processing questions.
7. Where might we encounter 3×3 matrices in real life?
8. Say: "Now that we understand the parts and structure of a 3×3 matrix, we'll explore how to find its determinant using two methods

B.2. Discussing the Concept

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

Methods on Determining of Determinants of a 3 X 3 Matrices
Method 1**:**
Expansion by Minors and Cofactors (The Universal Method)

1. Introduce the concept of a minor: Explain that for every element in a matrix, there is a "minor," which is the determinant of the

Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING

1. Present the two methods of finding the determinant of 3x3 matrices.
2. Guide the learners in to find the determinant of the matrix using the two methods.

smaller matrix left over when you cross out that element's row and column.

2. Explain cofactors: Introduce the concept that each minor is multiplied by a "cofactor," which is either +1 or -1, depending on the element's position. Display the alternating sign grid for a 3x3 matrix:

Cofactor Expansions

A recursive formula must have a starting point. For cofactor expansions, the starting point is the case of matrices. The definition of determinant directly implies that 1x1 matrices.

To describe cofactor expansions, we need to introduce some notation.

$$\det(a) = a$$

Definition : Minor and Cofactor

Let be an *nxn* matrix.

1. The (i.j) ***minor***, denoted is $\mathbf{A_{ij}}$, is the (n-1)x(n-1) matrix obtained from A by deleting the *i*th row and the *j*th column.
2. The (i.j) ***cofactor*** $\mathbf{C_{ij}}$ is defined in terms of the minor by

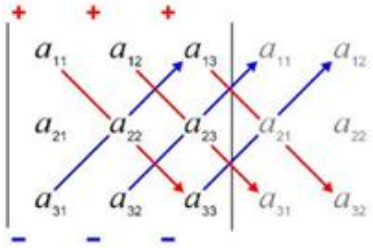
$$\mathbf{C_{ij} = (-1)^{i+j} \det(A_{ij})}$$

Note that the signs of the cofactors follow a "checkerboard pattern." Namely, $(-1)^{i+j}$ is pictured in this matrix:

$$[+ - + - + - + - +] + - + - + - + - +$$

Method 2: Sarrus' Rule (The Shortcut Method for 3x3)

3. Explain the setup: Show learners how to rewrite the first two columns of the matrix to the right of the original matrix.



4. Demonstrate the diagonal products:

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{bmatrix}$$

3. Facilitate the discussion, then ask:
   a. Which method was faster?
   b. Which method helped you understand the process better?
   c. Did both groups get the same determinant value?
4. Conclude the activity bay saying:

"Both methods lead to the same result, but each has its own advantage. Sarrus' Rule is quick and visual, while expansion by minors ensures accuracy and deeper understanding."

For

Compute A_{23} and C_{23}.

$$A_{23} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 7 & 8 \end{pmatrix} \qquad C_{23} = (-1)^{2+3} \det \begin{pmatrix} 1 & 2 \\ 7 & 8 \end{pmatrix} = (-1)(-6) = 6$$

Instructions:

1. Review the two methods
   Briefly recap Sarrus' Rule and Cofactor Expansion using one simple example per method.
2. Present two 3×3 matrices
   Choose one "clean" matrix suitable for Sarrus' Rule and one more complex matrix ideal for Cofactor Expansion.
3. Guide learners in comparison
   Ask learners to examine the matrices and discuss which method they think will work better and why.
4. Facilitate computation
   Allow learners to compute determinants using the method they chose *first*, then require them to verify using the *other* method.
5. Lead a reflection discussion
   Ask questions such as:
   a. "Which method was faster for you?"
   b. "Which method felt more reliable or less confusing?"
   c. "When might accuracy matter more than speed?"
6. Summarize the core idea
   Emphasize that Sarrus is fast but limited, while Cofactor

	5. Forward Diagonals (Add): Trace the three downward diagonal paths with colored markers and explain that the products of the elements along these paths are added. 6. Backward Diagonals (Subtract): Trace the three upward diagonal paths with a different color and explain that the products of the elements along these paths are subtracted. 7. State the caveat: Explicitly state that this rule only works for 3x3 matrices **Guided Activity:** **Find the determinant of:** 1. $\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{bmatrix}$ **Solution:** $\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{bmatrix}$ *using Method* 1 **Solution** We make the somewhat arbitrary choice to expand along the first row. $A_{11} = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$ $C_{11} = +\det\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} = 4$ $A_{12} = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix}$ $C_{12} = -\det\begin{pmatrix} -1 & 1 \\ -2 & 3 \end{pmatrix} = 1$ $A_{13} = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -2 & 2 \end{pmatrix}$ $C_{13} = +\det\begin{pmatrix} -1 & 2 \\ -2 & 2 \end{pmatrix} = 2.$ Det(A) = $a_{11}C_{11}+a_{12}C_{12}+a_{13}C_{13}$ = = (2)(4)+(1)(1)+(3)(2) = 15 **Find the determinant of** $\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{bmatrix}$ *using Method 2* Solution:	Expansion works for all matrix sizes, making it essential in higher mathematics.

Step 1: Rewrite the first two columns after the matrix

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ -2 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ -1 & 2 \\ -2 & 2 \end{bmatrix}$$

Step 2. Multiply the diagonals going down (left to right)

a. (2)(2)(3)=12
b. (1)(1)(−2)=−2
c. (3)(−1)(2)=−6

Sum of downward diagonals = 12+(−2)+(−6)=412 + (-2) + (-6) = 412+(−2)+(−6)=4

Step 3. Multiply the diagonals going up (right to left)

a. (3)(2)(−2)=−12
b. (2)(1)(2)=4
c. (3)(−1)(1)=−3

Sum of upward diagonals = -12 + 4 + (-3) = -11

Step 4. Subtract

Det(A)=(downward sum)−(upward sum)

Det(A)=4 - (-11)=15

B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)

Activity B.3.
PRACTICING LEARNED SKILLS: MASTERING THE 3X3 METHODS

Before facilitating this activity, ensure that all learners have a copy of the LAS.

1. Divide the class into small groups (3–5 learners per group).
2. Assign each group one or more 3×3 matrices of varying difficulty.
3. Designate:
   a. Half of the groups to solve the determinants using Sarrus' Rule.
   b. The other half to solve the *same determinants* using Cofactor Expansion (any row or column).
4. After completing the computation, pair or rotate groups so that each group can compare results with a group that used the *other method*.
5. Instruct learners to:
   a. Check if both methods produce the same determinant value.
   b. Discuss possible reasons if answers differ.
6. Move around the room to monitor progress, guide discussions, and correct misconceptions.

<table>
<tr><td></td><td colspan="2">7. Facilitate a short class reflection focusing on:
• Accuracy of results
• Efficiency of each method
• Common sources of error</td></tr>
<tr><td rowspan="2">C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
1. Examine the real-life scenarios provided by your teacher (traffic maps, seating arrangements, digital images, sales tables, weather logs, agricultural data, etc.).
2. Choose one scenario that interests you the most. then
• Identify the 3×3 matrix in the situation.
• Write the matrix clearly.
• Compute the determinant using either Sarrus' Rule or Cofactor Expansion.
3. Explain what the determinant tells you about the real-world scenario
4. Identify the matrix in the scenario by locating the rows, columns, and entries.
5. Compute the determinant of the matrix.
6. Work with a partner or small group to discuss your findings.
7. Be ready to share your output with the class.

Scenario 1. Jeepney Route Grid (Transport System)
The matrix below shows estimated travel times (in minutes) between three terminals.
$$T = \begin{bmatrix} 0 & 12 & 18 \\ 10 & 0 & 20 \\ 15 & 14 & 0 \end{bmatrix}$$
Tasks:
a. Compute det(T).
b. What does the sign/value tell you about route consistency?
c. How can determinants help in planning traffic systems?
Solution:
Interpretation:

Scenario 2.Barangay Basketball Stats
A local sports coordinator recorded the game performance of three players:</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

1. Present 2–4 real-world visuals such as:
a. Traffic flow tables
b. Seating arrangement grids
c. Pixel brightness images
c. Store sales tables
d. Weather data logs
d. 3×3 production tables
2. Guide learners to:
a. Identify the matrix structure
b. Recognize rows, columns, and numerical patterns
c. Understand what the determinant might represent
3. Encourage learners to:
a. Work collaboratively
b. Explain their reasoning
c. Compare interpretations with others
4. Move around the room to assist with matrix formation and determinant computation.
5. Facilitate sharing by letting selected groups present:
a. Their matrix
b. Their determinant
c. Their interpretation
6. Conclude by emphasizing:
a. How matrices appear naturally in organized information
b. How determinants help analyze relationships (e.g., changes, trends, independence, proportionality)
c. How these mathematical tools support decision-making and problem-solving in real life.</td></tr>
</table>

$$S = \begin{bmatrix} 18 & 9 & 4 \\ 22 & 11 & 6 \\ 15 & 7 & 3 \end{bmatrix}$$

Columns represent: **Points – Rebounds – Assists**

Tasks:

a. Compute det(S).

b. What does the determinant suggest about performance patterns?

c. Why is this helpful for team analysis?

Solution:

Interpretation:

Scenario 3. Sari-Sari Store: 3-Day Sales Matrix

$$M = \begin{bmatrix} 20 & 25 & 22 \\ 10 & 12 & 13 \\ 15 & 18 & 17 \end{bmatrix}$$

Rows: Rice, Canned Goods, Drinks

Columns: Mon, Tue, Wed

Tasks:

a. Compute det(M).

b. What does the determinant tell you about sales progression?

c. Why is determinant analysis useful in business?

Solution:

Interpretation:

Scenario 4. Agriculture: Rice Production (3 Provinces × 3 Months)

$$R = \begin{bmatrix} 120 & 130 & 140 \\ 100 & 110 & 115 \\ 150 & 145 & 160 \end{bmatrix}$$

Tasks:

a. Compute det(R).

b. What does the determinant indicate about regional production changes?

c. How can this help in agricultural planning?

Solution:

Interpretation:

Tasks:

a. Compute det(S).

b. What does the determinant suggest about performance

	patterns? c. Why is this helpful for team analysis? **Solution:** **Interpretation:** **Scenario 3. Sari-Sari Store: 3-Day Sales Matrix** $M = \begin{bmatrix} 20 & 25 & 22 \\ 10 & 12 & 13 \\ 15 & 18 & 17 \end{bmatrix}$ Rows: Rice, Canned Goods, Drinks Columns: Mon, Tue, Wed **Tasks:** a. Compute det(M). b. What does the determinant tell you about sales progression? c. Why is determinant analysis useful in business? **Solution:** **Interpretation:** **Scenario 4 — Agriculture: Rice Production (3 Provinces × 3 Months)** $R = \begin{bmatrix} 120 & 130 & 140 \\ 100 & 110 & 115 \\ 150 & 145 & 160 \end{bmatrix}$ **Tasks:** a. Compute det(R). b. What does the determinant indicate about regional production changes? c. How can this help in agricultural planning? **Solution:** **Interpretation:**	
	C.2. Making Generalization	
	Activity C.2. WRAPPING-UP THE LESSON **Instructions** 1. Work with your group and reflect on the determinant activities you completed. 2. Discuss how Sarrus' Rule and Cofactor Expansion compare. 3. As a group, complete each generalization statement in one clear sentence.	**Activity C.2. WRAPPING-UP THE LESSON** 1. Ask learners to remain in their previous small groups. 2. Provide each group with a short generalization task sheet or display the guide prompts on the board. 3. Explain that the goal is not to compute, but to draw conclusions based on what they observed while solving determinants using different methods.

	4. Be ready to explain *why* your group agrees on each conclusion. **Generalization Prompts (Group Output)** **Complete each statement:** 1. Regardless of the method used, the value of a 3×3 determinant _____. 2. Sarrus' Rule is most useful when _____. 3. Cofactor Expansion is more practical when _____. 4. Errors in determinant computation usually occur when _____. 5. A determinant equal to zero means that _____. 6. The most important idea we learned about determinants is _____.	4. Allow 5 minutes of group discussion. 5. Instruct each group to agree on one clear general statement for each prompt. 6. Call on selected groups to share their responses. 7. Write correct and insightful generalizations on the board and refine them with the class.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. **ASSESSING LEARNING OUTCOMES: CRACK THE CODE OF THE 3×3 DETERMINANT** Before facilitating this activity, ensure that all learners have a **copy** of the LAS. 1**.** Remind learners that this activity serves as a checkpoint for understanding of 3×3 determinants. 2. Encourage learners to show full solutions, including diagonals or cofactor steps. 3. Circulate around the room and look for: a. Correct application of rules b. Logical explanation of determinant meaning c. Ability to interpret real-world relevance 4. After completion, ask volunteers to share: a. Their computational strategies b. How they decided which method to use c. How determinants helped them interpret data 5. Provide immediate feedback to address any misconceptions. 6. Use the results to inform your next lesson (e.g., Cramer's Rule, Inverses).	
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.4. **EXTENDING AND REINFORCING LEARNING: CHOOSE THE BEST METHOD** Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Organize learners into small groups (3–4 members). 2. Provide each group with two 3×3 matrices that differ in structure (e.g., presence of zeros, repeated entries). 3. Explain that learners must:	

	a. Decide which method (Sarrus' Rule or Cofactor Expansion) is more appropriate for each matrix. b. Justify their choice before computing. 4. Emphasize that method selection and reasoning are as important as the final answer. 5. Circulate around the classroom to: a. Guide group discussions b. Ask probing questions c. Correct misconceptions when necessary 6. Call on selected groups for the presentation. 7. Summarize key takeaways on the board regarding efficient method selection.

III. CONTENT	**Lesson 3.7. MATRIX INVERTIBILITY**	
IV. OBJECTIVES	**At the end of the lesson, learners should be able to:** **1. state the condition for matrix invertibility; and** **2. test if the given 2×2and 3x3 matrices are invertible using determinants**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. **LEVELING LEARNERS READINESS: ZERO OR NON ZERO?** **Instructions:** 1. Observe the given matrices and find their determinant values. **Matrix A (2×2)** $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ **Matrix B (2×2)** $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ **Matrix C (3×3)** $C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 0 & 4 \end{bmatrix}$ 2. Identify which matrices have zero and nonzero determinants. 3. Describe any pattern you notice in matrices with determinant zero. 4. Participate in the class discussion by answering the questions.	**Activity A.1.** **LEVELING LEARNERS READINESS: ZERO OR NON ZERO?** 1. Present the following matrices and ask the learners find determinant. **Matrix A (2×2)** $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ **det** $= -1$ **Matrix B (2×2)** $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ **det** $= 0$ **Matrix C (3×3)** $C = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 3 & 0 & 4 \end{bmatrix}$ **det** $\neq 0$ 2. Ask the learners: a. Which matrices have nonzero determinants? b. Which matrix has a determinant equal to zero? c. What do you observe about the rows or columns of the matrix with determinant zero?

		3. Encourage learners to relate their answers to prior lessons on determinants. 4. Emphasize that both 2×2 and 3×3 matrices follow the same determinant principle.
	A.2. Establishing the Purpose of the Lesson	
	Activity A. 2. APPRECIATING LESSON RELEVANCE **Instructions**: 1. Observe the following everyday actions: a. Mixing two colors of paint. b. Encrypting a message using a code. c. Rotating or resizing an image on a computer. d. Solving a system of equations to find unknowns. 2. Discuss in groups: Which of these actions can be undone to recover the original state? Which cannot? 3. Classify each situation as *reversible* or *non-reversible* and explain your reasoning. 4. Share your group's insights with the class. 5. Reflect: What makes some processes reversible while others are not? How might mathematics help us describe this idea? **Processing Questions**: 1. What patterns do you notice among reversible processes? 2. How can we represent such processes mathematically? 3. What might prevent a process from being reversed?	**Activity A. 2. APPRECIATING LESSON RELEVANCE** 1. Encourage learners to connect "reversibility" with mathematical operations they already know (e.g., addition/subtraction, multiplication/division). 2. Use their examples to introduce the idea that some matrices allow reversal while others do not—without naming the concept yet. 3. Conclude by saying: "In our next activity, we'll explore how to determine whether a matrix can be reversed using a special property."
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Instructions:** 1. Study the given matrices below. Each matrix shows a different pattern of rows — independent, proportional, identical, or containing zeros. Independent Rows $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ Proportional rows $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ Zero row	**Activity B.1. EXPLORING KEY CONCEPTS** 1. Provide learners with **4 matrices** including examples with: a. Identical rows b. A zero row or zero column c. Proportional rows d. Independent rows 2. Guide learners to compute the determinant of each matrix and record their results in the table. 3. Encourage collaborative discussion as learners identify patterns among matrices with zero determinants.

	$C = \begin{bmatrix} 4 & -1 & 2 \\ 0 & 0 & 0 \\ 3 & 5 & 1 \end{bmatrix}$ Identical rows $D = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 4 \end{bmatrix}$ 2. Observe the relationships among the rows. 3. Compute the determinant to identify whether it is a zero or non zero. 4. Predict whether the matrix is invertible or not invertible. 5. Record your answers in the table below:	4. Circulate and prompt learners to justify their observations using properties of determinants (e.g., effect of identical or proportional rows). 5. After learners complete the table, lead a reflection discussion using the same question: What patterns did you notice in non-invertible matrices? 6. Summarize key takeaways emphasizing that matrices with zero determinants are **non-invertible** due to linear dependence among rows.

Given	Expected determinant	Zero or non-zero	Invertible? Yes/No
Indepedent Rows $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$			
Proportional rows $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$			
Zero row $C = \begin{bmatrix} 4 & -1 & 2 \\ 0 & 0 & 0 \\ 3 & 5 & 1 \end{bmatrix}$			
Identical rows $D = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 4 \end{bmatrix}$			

After completing the table, answer the reflection questions:

	• What patterns did you notice in non-invertible matrices? • How does the determinant confirm your observations?	
	B.2. Discussing the Concept	
	Activity B.2. **DEEPENING CONCEPTUAL UNDERSTANDING:** **WHAT MAKES A MATRIX INVERTIBLE?** **Instructions:** 1. Recall what you discovered in Activity B.1 about matrices with proportional, identical, or zero rows. 2. Think about how those patterns affected the determinant and invertibility. 3. Examine the matrices. $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ 4. Compute the determinant. 5. Analyze the results. a. Is the determinant zero or non-zero? b. Is the matrix invertible? Why?	**Activity B.2.** **DEEPENING CONCEPTUAL UNDERSTANDING:** **WHAT MAKES A MATRIX INVERTIBLE?** **1. Introduce the Core Idea** Explain that the **determinant** tells us whether a matrix is **invertible** or **not invertible**. a. A matrix is **invertible (non-singular)** if $det\ (A) \neq 0$ b. A matrix is **not invertible (singular)** if $det\ (A) = 0$ This rule applies to **all square matrices** (2×2, 3×3, 4×4, etc.) **2. Connect to Structure** Emphasize that the determinant reflects **row/column independence**: a. If all rows/columns give **independent information**, the determinant is **non-zero**, so the matrix has an inverse. b. If any row/column is: • identical • proportional • dependent • zero the determinant becomes ***zero***, and the matrix is **not invertible**. **Show Simple Examples** **Example 1. 2×2 Case** $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $det\ (A) = 3(4) - 5(2) = 12 - 10 = 2$ Because the determinant is **2 (not zero)**, ***A is invertible.*** **Example 2. 3×3 Case**

		$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ Compute determinant (learners can use Sarrus or Cofactor Expansion). Observation: Rows are **multiples**, so determinant = **0**.**B is NOT invertible.** **4. Connect to Real-Life Meaning** Explain: a. If the determinant is zero, the matrix **cannot be reversed**. In real-world applications (systems of equations, graphics, cryptography), b. invertibility determines whether a solution or transformation is possible. **Provide this as an example:** In computer graphics, matrices perform transformations such as: a. rotation, b. reflection, c. scaling, d. stretching. **Why invertibility matters** An invertible matrix allows the computer to reverse a transformation. Example: a. Zooming in and then zooming out b. Rotating an image and rotating it back If the determinant is zero: a. the image may flatten into a line or point, b. information is lost, c. the original image cannot be recovered **Concept Summary (write on board)** a. A matrix is invertible ⇢ det ≠ 0 b. A matrix is not invertible ⇢ det = 0 c. Determinant helps test independence of rows/columns d. Invertibility determines if a system or transformation can be reversed
	B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)	

<table>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.
1. Prepare 6 matrices (3 of them invertible, 3 not invertible).
Include examples with: independent rows (invertible)
a. proportional rows (not invertible)
b. zero rows or columns (not invertible) mixed numbers for computation practice
2. Remind leaners of determinant rules:
a. $ad - bc$for 2×2
b. Sarrus' Rule or cofactor expansion for 3×3Let learners compute independently first.
3. Facilitate partner or group checking for mathematical communication.
4. Call learners to explain selected solutions on the board.
5. Emphasize the key idea: "A matrix is invertible if and only if det ≠ 0."
6. Clarify errors such as:
7. Incorrect minor signs (cofactor errors)

a. mixing up forward/backward diagonals
b. forgetting to compare determinant to zero</td></tr>
<tr><td rowspan="2">C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions
1. Read each real-life scenario provided below.
2. Choose ONE scenario that interests you the most.
3. Identify the 3×3 matrix inherent in that situation (if not directly shown).
4. Compute its determinant.
5. Determine whether the matrix is invertible or not invertible.
6. Explain what the determinant means in the context of the scenario.
7. Write your answers clearly and be ready to share your explanation with the class.

Real-Life Scenarios (Learners Pick One)
Scenario 1. Traffic Flow & Route Prediction
A city uses a 3×3 travel-time matrix to estimate commuter flow between three major intersections.
If the determinant is non-zero, the system can predict unique traffic adjustments.</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

1. Present the scenarios below using visuals or short descriptions.
2. Encourage learners to identify which part of the situation forms a matrix.
3. Ask learners to compute determinants and discuss their interpretations.
4. Highlight that invertibility = unique, reversible, or reliable outcomes, while non-invertibility = dependency, collapse, or f failure of the model.
5. Allow groups to compare how determinants affect different real-world systems.</td></tr>
</table>

If it is zero, the model collapses and traffic behavior cannot be uniquely predicted.
Matrix:

$$T = \begin{bmatrix} 0 & 12 & 18 \\ 10 & 0 & 20 \\ 15 & 14 & 0 \end{bmatrix}$$

Scenario 2. Image Transformations (Computer Graphics)
A 3×3 transformation matrix is used to rotate and scale a digital object.
A non-invertible matrix means the transformation *cannot* be reversed, distorting the image permanently.
Matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

Scenario 3. Cryptography (Encoding & Decoding Messages)
A message is encoded using a 3×3 key matrix.
To decode the message, the matrix must be invertible.
If the determinant = 0 → the message cannot be recovered.
Matrix:

$$K = \begin{bmatrix} 4 & 7 & 2 \\ 3 & 5 & 1 \\ 2 & 6 & 3 \end{bmatrix}$$

Scenario 4. Economic Forecasting
Economists analyze how GDP, inflation, and employment interact using a 3×3 matrix.
If the determinant is zero, the indicators are dependent and forecasts will be unreliable.
Matrix:

$$E = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 4 & 1 & 5 \end{bmatrix}$$

Scenario 5. Robot Arm Movement (3D Orientation Matrix)
A robot uses a 3×3 orientation matrix to move in 3D space.
If the matrix is NOT invertible, the robot loses degrees of freedom and cannot reverse or correct movements.
Matrix:

$$R = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 4 \\ 2 & 5 & 1 \end{bmatrix}$$

Learner Output Format

	Chosen Scenario: _____ 1. Matrix Identified: [_ _ _ _ _ _ _ _ _] 2. Determinant: _____ 3. Invertible? ☐ Yes ☐ No 4. Interpretation (Meaning in the scenario):	
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Instructions.** 1. Look back at everything you did today—predicting invertibility, computing determinants, identifying patterns, and applying determinants to real-life situations. 2. Using your understanding, complete each generalization statement below. Write 1–2 clear sentences per item. **Complete the statements:** a. A matrix is invertible when… b. A matrix becomes non-invertible when… c. The determinant helps us determine invertibility because… d. From the examples we used, I noticed that matrices with a determinant of zero… e. In real-life situations, having an invertible matrix is important because…	**Activity C.2. WRAPPING UP THE LESSON** 1. Tell learners that the goal of this activity is to help them form conclusions from patterns discovered during the lesson. 2. Allow 3–5 minutes of quiet reflection before writing. 3. Encourage LEARNERs to use their own words, avoiding memorized textbook definitions. 4. Ask a few volunteers to share their answers. 5. Highlight key ideas such as: a. Determinant ≠ 0 → matrix is invertible b. Determinant = 0 → matrix is not invertible 6. Invertibility relates to unique solutions and reversible transformations 7. Clarify any misunderstandings and refine ideas as a class.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Tell learners this activity serves as the final understanding check before moving to Cramer's Rule or inverse matrices. 2. Remind them that correct computation and reasoning are both important. 3. Circulate the room to check solution steps, especially: a. Sarrus' Rule application b. Cofactor sign errors	

	c. Distinguishing determinant = 0 from ≠ 0 4. Ask a few learners to present: a. Their determinant computation b. Their reasoning for invertible/not invertible c. Their real-life interpretation 5. Provide corrective feedback and emphasize: "A matrix is invertible if and only if det ≠ 0." 6. Use the results to identify who needs more support before moving to the next topic.
	C.4. Additional Activities
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a **copy** of the LAS. 1. Begin by recalling the concept of determinant and its role in identifying matrix invertibility. 2. Encourage learners to select authentic, relatable contexts, such as sales data, image scaling, or coded message systems—to embed their matrices meaningfully. 3. Guide learners in constructing their matrices and remind them to: a. Label rows and columns clearly according to their scenario. b. Show all computation steps for det(S). 4. Circulate the room to check for correct application of Sarrus' Rule or cofactor expansion and proper interpretation of results. 5. Ask a few learners to present their work, emphasizing: a. Their determinant computation process. b. Their reasoning for invertibility or non-invertibility. c. Their real-life interpretation of what the determinant implies. 6. Provide feedback highlighting both mathematical accuracy and conceptual understanding, how the determinant connects to reversibility or independence in real-world systems. 7. Conclude by synthesizing key insights: a nonzero determinant signifies a reversible or independent process, while a zero determinant indicates dependency or loss of information.

III. CONTENT	**Lesson 3.8. CRAMER'S RULE**	
IV. OBJECTIVES	**At the end of the lesson, learners should be able to apply Cramer's Rule to solve 2×2 and 3×3 systems of linear equations.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITAION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS **Instructions**:	**Activity A.1. LEVELING LEARNER READINESS** 1. Introduce the activity as a fun, fast-paced warm-up to activate learners' prior knowledge about determinants. 2. Clarify the rules and scoring:

	1. Team up with 3–4 classmates. Your goal is to create matrices whose determinants equal zero — as quickly and accurately as you can! 2. Choose your challenge level: 🟩 2×2 matrix → 1 point 🟧 3×3 matrix → 2 points 3. You have 10 minutes to create as many valid zero-determinant matrices as possible. 4. For each matrix, explain why its determinant will be zero *before* you compute it. 5. Present one matrix to the class and defend your reasoning. 6. After the challenge, reflect on your process by answering: a. What patterns helped you predict a zero determinant? b. How did your group decide which numbers to use? c. What did you notice about row or column relationships? d. How might this idea connect to whether a matrix is invertible?	a. Only 2×2 and 3×3 matrices are allowed. b. 2×2 = 1 point, 3×3 = 2 points. c. Time limit: 10 minutes. 3. Encourage reasoning over computation: Learners should justify zero determinants through logical patterns, not full calculations. 4. Observe and highlight reasoning indicators**:** a. Proportional rows or columns b. Identical rows or columns c. Rows or columns containing all zeros 5. Facilitate reflection: Use the process questions to help learners articulate their strategies and connect them to matrix invertibility. 6. Bridge to next lesson: Conclude by explaining that recognizing when determinants become zero prepares learners to apply Cramer's Rule for solving systems of linear equations in the next activity.
	A.2. Establishing the Purpose of the Lesson	
	Activity A. 2. **APPRECIATING LESSON RELEVANCE:** **REAL-LIFE PROBLEMS FOR CRAMER'S RULE** **Instructions** 1. Observe and discuss the following real-life situations with your partner or group: **a. Business Pricing Problem (2×2 System)** A school canteen sells a combo meal consisting of 1 sandwich and 1 juice for ₱85. A second combo has 2 sandwiches and 1 juice for ₱130. **b. Mixture Problem (2×2 System)** A laboratory technician is mixing two chemical solutions: • Solution A has 20% concentration • Solution B has 50% concentration She wants to prepare 100 mL of a 34% concentration **c. Traffic Flow Problem (3×3 System)**	**Activity A. 2.** **APPRECIATING LESSON RELEVANCE:** **REAL-LIFE PROBLEMS FOR CRAMER'S RULE** 1. Present four (4) real-life problems where solving a system of equations is essential 2. Guide the discussion: 3. Ask the following questions to help learners intuitively grasp the idea of uniqueness and dependency before formalizing it mathematically. How many unknowns do we have? a. Can we represent these relationships using equations? b. Why might we need one exact solution? c. What happens if the relationships don't allow a unique solution?" 4. Facilitate group work: a. Have learners choose one scenario and express it as a system of equations. b. Encourage reasoning about what makes a system c. c. solvable or unsolvable. 5. Circulate to check that learners correctly identify variables and relationships.

	At a 3-way intersection, traffic engineers observe these conditions: ● Total cars entering Road A, Road B, and Road C is 420 cars/hour. ● Twice the number of cars entering Road A plus the number entering Road B minus the number entering Road C is 300. ● The number entering Road A plus the number entering Road B plus twice the number entering Road C is 520. **d. Budget Allocation Problem (3×3 System)** A school allocates its quarterly budget among Science, Math, and ICT departments. The following relationships are known: a. The total budget allotted is ₱300,000. b. Science receives ₱20,000 more than Math. c. ICT receives twice as much as Science. 2. Choose one scenario and work with a partner to: a. Identify the unknowns (what you need to find). b. Describe how the situation could be represented using equations. c. Discuss why it's important to have one exact solution in that situation. d. Predict what might happen if the relationships between quantities don't allow a unique solution. 3. Reflect and connect: a. Why do some problems require one exact answer while others allow many possibilities? b. What clues in your equations suggest whether a unique solution exists? 4. Optional Challenge (for advanced learners):	6. **Lead the reflection:** After presentations, ask process questions to deepen understanding: a. What clues in your equations show whether a unique solution exists? b. Why do real-life problems often require one exact solution? 7. How might mathematicians design a method to find that solution efficiently? (This primes learners for the upcoming introduction of **Cramer's Rule**.) 8. **Differentiate learning:** a. For learners needing support: use **2-variable** examples with guided reasoning. b. For advanced learners: extend to **3-variable** examples and introduce matrix representation informally. 9. Encourage visual learners to use tables or diagrams to represent relationships. 10. **Transition to next lesson:** Conclude by emphasizing that today's activity shows *why* unique solutions matter. In the next lesson, learners will discover a systematic way—**Cramer's Rule**—to find those solutions using determinants.

	5. Extend your chosen scenario to include three variables instead of two. 6. Sketch how you might organize the information in a table or matrix to make solving easier.	
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. **EXPLORING KEY CONCEPTS; CRAMER'S RULE APPROACHES** **Option 1. First Approach** **Instructions** 1. Then, watch how your teacher uses Cramer's Rule to find the solution step-by-step. $2x+3y=5$ $4x-y=7$ 2. Copy the matrices shown during the demonstration into your notebook — the coefficient matrix A, and the determinants Dx and Dy. 3. Observe how each determinant is computed and think about what changes when columns are replaced. 4. Predict the values of x and y before your teacher reveals the final answer. **Option 2. Rule-First Approach** **Instructions**: 1. Study Cramer's Rule as your teacher explains it. Then, apply the rule to solve the system below, showing all determinant computations clearly. $2x+3y=5$ $4x-y=7$ 2. Write your solution neatly and check if your computed values satisfy both equations. *3.* Reflect on how replacing columns in the matrix helps isolate each variable.	**Activity B.1.** **EXPLORING KEY CONCEPTS; CRAMER'S RULE APPROACHES** **Option 1. Activity-First Approach** 1. Present the system of equations on the board. $2x+3y=5$ $4x-y=7$ 2. Ask learners to attempt solving it before introducing Cramer's Rule. 3. Demonstrate how to form the coefficient matrix A, then replace columns to obtain Dx and Dy. 4. Identify the coefficient matrix and label it clearly: $D = \begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$ to find det (D) 5. Replace the x-column with the constants to find D_x. 6. Replace the y-column with the constants to find D_y 7. Explain the formula $ad - bc$ step-by-step. 8. Emphasize that Cramer's Rule only works if det(A) ≠ 0. 9. Compute each determinant and show how $x=Dx/D$ and $y=Dy/D$. 10. Compute each determinant and show how to substitute them into: $x = \frac{D_x}{D}, y = \frac{D_y}{D}$ 11. Repeat with a 3×3 system, if needed, using Sarrus' Rule or cofactor expansion. 12. Highlight key ideas during the demonstration: 13. Determinants tell us if the system has a unique solution. 14. Replacing columns isolates variables. 15. Cramer's Rule is systematic and avoids long **Option 2. Rule-First Approach** 1. Introduce the General Form of an Augmented Matrix

		For a 2 x 2 system of linear equations: $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ the augmented matrix is written as: $\left[\begin{array}{cc\|c} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array}\right]$ For a 3 × 3system: $\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$ the augmented matrix is: $\left[\begin{array}{ccc\|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$ 2. Present the general form of Cramer's Rule for a 2×2 system: 1. $x = \frac{D_x}{D}, y = \frac{D_y}{D}$ Where D = $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ $D_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}$ $D_y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$ 3. Model determinant computation for D, D_x, and D_y, given these equations: 2x+3y=5 4x−y=7 the augmented matrix is written as $\left[\begin{array}{cc\|c} 2 & 3 & 5 \\ 4 & -1 & 7 \end{array}\right]$ D = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ $D_x = \begin{bmatrix} 5 & 3 \\ 7 & -1 \end{bmatrix}$ $D_y = \begin{bmatrix} 2 & 5 \\ 4 & 7 \end{bmatrix}$ 4. Reinforce the relationship $x = D_x/D$ and $y = \frac{D_y}{D}$through guided questioning. 5. Let learners compute and verify their own solutions.:

		$x = \frac{13}{7}, y = \frac{3}{7}$
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Instructions**: 1. Form groups of 3–4 members. 2. Your teacher will assign several systems of equations (2×2 and 3×3). **Practice Set**: a. x+2y=8 3x – y = 1 b. 2x −y+3z=12 x+4y−z=53 x+2y+2z=14 c. x +y+z=6 2x−y+3z=14 3x+4y−z=5 3. For each system: • Write the coefficient matrix A and the augmented matrix [A | B]. • Compute the determinant *D*. • Predict whether the system has one, none, or infinitely many solutions based on *D*: If *D* ≠ *0*: The system has one unique solution. If *D* = *0*: The system has no solution or infinitely many solutions — check consistency using [A | B]. 4. Use Cramer's Rule to find *x*, *y*, and *z* (if 3×3). 5. Compare your prediction with the actual result. 6. Record your computations neatly in your notebook. 7. After solving, answer the questions below. **Interactive Extension**: Play "Determinant Dash!" Each group races to compute *D* for a randomly assigned system. Shout "Unique!", "None!", or "Infinite!" depending on your result. The fastest correct group earns a point! **Processing Questions**: 1. What does the determinant tell you about the solvability of a system?	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** 1. Preparation: a. Prepare 4 systems of equations (2×2 and 3×3) with varying determinant values. b. Write each system clearly on the board or slides. c. Provide learners with blank matrix templates for computation. 2. Guided Demonstration: a. Begin with a simple 2×2 system. b. Show how to form the coefficient matrix A and augmented matrix [A | B]. c. Compute *D*, *Dx*, *Dy*, and *Dz* (if 3×3). d. Model how to predict solution type: • Ask learners: "What does the determinant tell us?" • Guide them to reason that: ○ If *D* ≠ *0*, the system is consistent and independent → one unique solution. ○ If *D* = *0*, the system is either inconsistent (no solution) or dependent (infinitely many solutions). e. Demonstrate checking consistency by comparing the rank of A and [A | B]. f. Emphasize the relationship between determinant values and system behavior. g. Model substitution to verify computed solutions. 3. Group Facilitation: a. Assign each group one system to solve. b. Circulate and check determinant computations. c. Encourage learners to explain their reasoning aloud. d. Use color-coded matrices or digital tools (Desmos, GeoGebra, Excel) for visualization. 4. Debrief and Reflection: a. Lead a short discussion comparing predictions and results. b. Ask learners to connect determinant values to system consistency.

<table>
<tr><td></td><td>2. How does replacing one column at a time isolate each variable?
3. Why does Cramer's Rule fail when $D = 0$?
4. Which step in computing determinants do you find most challenging?
5. How does the augmented matrix help visualize the entire system?</td><td>c. Reinforce that $D \neq 0$ indicates a unique solution, while $D = 0$ requires checking for consistency to distinguish between no and infinite solutions.</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3: PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.
1. Assign the systems below (2×2 and 3×3) for independent mastery practice.
2. Remind learners to compute:
det(A)
D_x, D_y for 2×2
D_x, D_y, D_z for 3×3
3. Encourage organized work (label each matrix clearly).
4. Move around to check computation accuracy and common errors (e.g., sign mistakes, wrong column replacement).
5. Ask 2–3 LEARNERs to present their full solution on the board.
6. Use the answer key for quick verification.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
1. Choose ONE real-life scenario from the list provided.
2. Assign a variable to the items or objects and represent them into an equation in the situation.
3. Translate the scenario into a system of linear equations (2×2 or 3×3).
4. Use Cramer's Rule to solve for the unknown values.
5. Interpret your answer in the context of the situation.
6. Share your solution and interpretation with a partner or small group.

Real-Life Application Scenarios
1. Pricing In A Café (2×2 System)
A café sells two combo meals:</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

1. Present 3–4 real-life situations that naturally translate into systems of linear equations.
2. Guide learners in identifying the unknowns, equations, and relationships in each scenario.
3. Remind learners to compute:
a. det(A)
b. Replaced determinants (Dx, Dy, Dz)
4. Encourage learners to explain what their answers mean in the real world.
5. Facilitate sharing and reinforce how mathematics helps solve real problems.</td></tr>
</table>

a. Combo A: 1 burger + 1 drink = ₱150

b. Combo B: 2 burgers + 1 drink = ₱230

Let:
x = price of 1 burger
y = price of 1 drink
System:

$$\{x + y = 150\ 2x + y = 230$$

Task: Use Cramer's Rule to find the price of each item.
Interpretation: What does this tell you about the store's pricing?

2. Chemical Mixing (2×2 System)

A chemist wants to mix two solutions:

a. Solution A: 30% concentration

b. Solution B: 60% concentration to create 200 mL of a 48% solution.

Let:
x = amount of Solution A
y = amount of Solution B
System:

$$\{x + y = 200\ 0.30x + 0.60y = 96$$

Task: How many mL of each solution are needed?

3. Traffic Distribution At An Intersection (3×3 System)

At a three-road intersection, engineers observe:

a. Total number of cars entering Roads A, B, and C = 600 cars/hour

b. Cars satisfy the relationship: 2A + B – C = 420

c. Another condition: A + B + 2C = 710

Let A, B, C = number of cars entering each road.
System:

$$\{A + B + C = 600\ 2A + B - C = 420\ A + B + 2C = 710$$

Task: Use Cramer's Rule to determine traffic volume on each road.

4. School Budgeting (3×3 System)

	A school divides a ₱500,000 fund among Science (S), Math (M), and ICT (I). Conditions: a. Total budget: S + M + I = 500,000 b. Science receives ₱30,000 more than Math → S = M + 30,000 c. ICT receives twice the amount of Science → I = 2S System: $\{S + M + I = 500000\ S - M = 30000\ I - 2S = 0$ Task: How much does each department receive?	
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Instructions:** 1. Review the activities you completed today (examples, demonstration, practice, and real-life applications). 2. Based on your understanding, complete each generalization prompt below using 1–2 meaningful sentences. 3. Use your own words—no copying from the board or the module. 4. Share your answers with a partner, then prepare to share one answer with the class. **Generalization Prompts:** 1. Cramer's Rule works when… 2. The determinant is important because… 3. Replacing one column at a time helps because… 4. A system has a unique solution when… 5. Using Cramer's Rule helps me understand that…	**Activity C.2. WRAPPING UP THE LESSON** 1. Ask learners to reflect silently for 2–3 minutes before writing. 2. Encourage them to use their own observations from the lesson rather than textbook definitions. 3. Facilitate a quick sharing session where volunteers read their generalizations. 4. Highlight key big ideas: a. Determinants determine the existence of a unique solution. b. Cramer's Rule isolates variables via column replacement. c. The method only works for independent systems. 5. Reinforce correct ideas and clarify misconceptions.
	C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)	
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS. 1. Explain that this activity is a self-assessment and an opportunity for learners to show mastery of Cramer's Rule.	

	2. Remind learners to show all steps, especially determinant calculations (Sarrus' Rule or cofactor expansion for 3×3). 3. Move around the classroom to: a. identify common computation errors, b. check correct usage of determinant formulas, and c. ensure proper application of Cramer's Rule. 4. After submission, invite 2–3 learners to demonstrate their solutions on the board. 5. Facilitate a short reflection: a. "How does det(A) affect whether Cramer's Rule works?" b. "What mistakes should we avoid when replacing columns?"
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS. Facilitate the activity to further enhance the learners' understanding. Ask learners to perform the activity on their LAS, and encourage advanced learners to assist their classmates who need remediation.
V. ASSESSMENT	**To evaluate learners' success in attaining the intended learning competencies, the assessment tools and strategies provided on the link in the table of contents can be utilized to measure understanding, skills, and application of concepts**
VI. REFLECTION	**To assess and evaluate the effectiveness of the instruction, as well as to identify challenges and plan for improvements in this unit, teachers are encouraged to answer the reflective questions provided in the link indicated in the table of contents**

UNIT 4. INTRODUCTION TO LINEAR PTOGRAMMING

I. LEARNING GOALS	
Content Standard	The learners demonstrate knowledge and understanding of the fundamental concepts of linear programming.
Performance Standard	By the end of the quarter, the learners are able to model real-world situations using matrices and matrix operations. They are also able to formulate linear programming models and solve them using either the graphical or simplex method.
Learning Competencies	The learners… 1. illustrate linear programming and its significance in decision making; 2. formulate real-world problems into linear programming models; 3. graph the feasible region in a Linear Programming Problem; 4. determine the optimal solutions to a Linear Programming Problem using the Corner Point Theorem; 5. solve linear programming problems of two variables by graphical method, with or without technology; 6. construct an initial simplex tableau from an LP problem; and 7. perform simplex iterations to obtain an optimal solution to an LP problem.
II. REFERENCES and MATERIALS	
Textbook and Modules	• GeeksforGeeks. "Graphical Solution of Linear Programming Problems." Accessed April 24, 2026 https://www.geeksforgeeks.org/maths/linear-programming/ • Lumen Learning. "*6.4 – Linear Inequalities and Systems of Linear Inequalities in Two Variables." Hunter College – MATH101.*

	Accessed April 24, 2026 https://courses.lumenlearning.com/cuny-hunter-collegealgebra/chapter/outcome-graphing-linear-equations/ • Reyes, J. A. & Reyes, M. A, (2025). Finite Mathematics 1. Unlimited Books Library Services and Publishing Inc., Manila
Materials	• Instructional Materials/ Slide deck presentation • Learning Activity Sheet/Printed worksheets • Graphing Board
EdTech Tool	• Scientific Calculator (optional) • GeoGebra
Use of Generative AI	In preparing this Learning Exemplar (LE), the author(s) used generative artificial intelligence (AI) tools to assist in organizing ideas, developing sample activities, and generating formative assessment items. The author(s) carefully reviewed, validated, and finalized all content. The author(s) take full responsibility for the content of this LE.
III. CONTENT	**Lesson 4.1 FUNDAMENTAL CONCEPTS OF LINEAR PROGRAMMING**
IV. OBJECTIVES	**At the end of the lesson, learners are will be able to:** **1. define linear programming (LP)) and its importance in decision-making;** **2. identify situations suitable for LP modeling;** **3. identify variables, write objective functions, and convert verbal conditions to inequalities; and** **4. formulate complete LP models, and create own LP problem scenario.**

V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge***	
	Activity A.1. **LEVELING LEARNER READINESS:** **LIMITED BUT NEEDED** **Instructions***:* 1. Read each real-life situation below. 2. Identify what needs to be decided and what constraints affect each decision. 3. Discuss your ideas with a partner. 4. Answer the processing questions after analyzing all situations. • A student has 5 hours and must allocate time for homework and online modules. • A catering service has 20 kg of chicken and must prepare two dishes. • A school club has limited funds to support two upcoming projects. **Processing Questions:** 1. What do these scenarios have in common?	**Activity A.1.** **LEVELING LEARNER READINESS:** **LIMITED BUT NEEDED** 1. Ask learners to form pairs. 2. Display or distribute the situations. 3. Instruct them: *"Your task is to identify the decisions to be made, the limitations present, and the idea of choosing the best possible option in each situation."* 4. Provide chart paper or notebooks for learners to record their ideas (optional). 5. After the discussion, ask 2–3 pairs to present their outputs. 6. Facilitate the processing questions and emphasize that all situations involve: • choices • limited resources • the goal of achieving the best possible outcome

<table>
<tr><td></td><td>2. What kind of decisions must be made?
3. Can these decisions be expressed using numbers or amounts?
4. What limitations or constraints exist?
5. What do we usually do when we want the best outcome, but we have limitations?
6. How does defining what is “best” (e.g., highest output, maximum benefit, or most efficient use of time) affect the decision-making process in each scenario?
7. Which scenario requires the most careful balancing of limitations and desired outcomes? Justify your answer.
8. What might happen if the decision-maker focuses only on one option and ignores the limitations? Provide a possible consequence.</td><td></td></tr>
<tr><td></td><td colspan="2">A.2. Establishing the Purpose of the Lesson</td></tr>
<tr><td></td><td>Activity A. 2.
APPRECIATING LESSON RELEVANCE:
BEST OUTCOMES
Instructions:
● Study the table below.
● Work with your seatmate to identify the relationship between the goals and limitations.
● Answer the guide questions in complete sentences.
<table><tr><th>Situation</th><th>Possible Goal</th><th>Limitation</th></tr><tr><td>Food stall production</td><td>Maximize income</td><td>Ingredients, time</td></tr><tr><td>Printing Shop</td><td>Minimize cost</td><td>Ink, workforce</td></tr><tr><td>Delivery Service</td><td>Shortest Route</td><td>Fuel, distance</td></tr></table>Process Questions:
1. What pattern do you observe among the goals?
2. What pattern do you observe among the limitations?
3. Do you think there is a systematic way to make the best decision?
4. How do these goals and limitations affect each other?</td><td>Activity A. 2.
APPRECIATING LESSON RELEVANCE:
BEST OUTCOMES
1. Display the table or provide printed copies.
2. Ask learners to work in pairs and study patterns across the examples.
3. Remind them to look for keywords such as “maximize,” “minimize,” and “limitations.”
4. Facilitate a discussion after pair work by asking:
• “What do these goals have in common?”
• “What limits the decisions in each case?”
5. Guide learners in articulating the lesson purpose:
“Linear Programming is a systematic way of making the best decision under constraints.”
6. Present the lesson objectives of the unit.</td></tr>
<tr><td>B. Instituting New Knowledge</td><td colspan="2">B.1. Presenting Examples (30 minutes)</td></tr>
<tr><td></td><td>Activity B.1. EXPLORING KEY CONCEPTS</td><td>Activity B.1. EXPLORING KEY CONCEPTS</td></tr>
</table>

	Part I. Analyze and Guess the Hidden Rule Instructions: ● Group by 4. ● Read the Lemonade Stand problem carefully. ● Discuss and answer all the questions as a group. ● Be ready to share your group's reasoning with the class. **Lemonade Stand** A student wants to sell lemonade and iced tea. They have 10 lemons and 8 tea bags available. Lemonade uses 2 lemons per cup, while iced tea uses 1 tea bag per cup. The student aims to earn the highest possible income. Lemonade sells for ₱15 per cup, while iced tea sells for ₱10 per cup. **Questions**: 1. What choices must the student make in this situation? 2. What factors limit how many cups can be sold? 3. Suppose: $x = number\ of\ cups\ of\ lemonade$ $y = number\ of\ cups\ of\ iced\ tea$ How can the limits in the situation be described using x and y? (Think in terms of "used must not exceed available.") 4. How can the student's goal of earning the highest possible income be represented using x and y? **Part II. Story Sorting: LP or Not?** 1. Your teacher will provide story cards or display different scenarios. 2. Work with your group to classify each scenario into the correct category. 3. Place each card under the appropriate heading. ❖ **Has a goal + limits (possible LP situation)** ❖ **Not an LP situation** Choosing a cellphone plan with limited budget. Deciding the fastest route to school. Picking what movie to watch.	**Part I. Analyze and Guess the Hidden Rule** 1. Assign learners into groups and display or distribute the problem. 2. Instruct them: "Your goal is to identify the choices, limitations, and objective in this situation." 3. Circulate to support groups by asking probing questions. 4. After 7–10 minutes, ask groups to share their answers. 5. Summarize the LP components that were implicitly discovered: ***Variables:*** *lemonade cups, iced tea cups* ***Constraints:*** *lemons, tea bags* ***Objective:*** *maximize profit* **Part 2. Story Sorting: LP or Not?** 1. Prepare two labeled areas in the classroom. 2. Distribute cards or display the scenarios on slides. 3. Ask groups to classify each situation accordingly. 4. Allow groups to briefly defend one of their classifications. 5. Clarify misconceptions (e.g., an optimization problem must have a clear, measurable goal and constraints). **Note:** Group will sort the scenarios into the following categories: Situations with a clear goal and constraints (potential Linear Programming problems) Situations without a clear goal or without mathematical constraints

Distributing relief goods across barangays with limited supply.	
Choosing how many cupcakes and cookies to bake to maximize income.	
Selecting clothes for a party.	

B.2. Discussing the Concept

Activity B.2.
DEEPENING CONCEPTUAL UNDERSTANDING: LINEAR PROGRAMMING AND ITS COMPONENTS

Part 1. Linear Programming and Its Components (Variables, Objective, Constraints)

Linear programming is defined as a type of optimization problem that involves maximizing or minimizing a linear objective function, subject to linear equality and inequality constraints.

Key Terms:
Decision Variables – unknown quantities that need to be determined in order to achieve the optimal solution.
Objective Function – a mathematical expression that represents the goal to be maximized or minimized.
Constraints – linear inequalities or equations that represent the limitations or restrictions that the decision variables must satisfy.

Play a short teacher-selected video or presentation introducing Linear Programming (optional).

Ask learners to share key points they learned. The teacher may list their responses on the board.

Example 1: Recognizing LP Situations
A farmer wants to plant corn and beans on 10 hectares of land and maximize profit.

Learners identify:

Activity B.2.
DEEPENING CONCEPTUAL UNDERSTANDING: LINEAR PROGRAMMING AND ITS COMPONENTS

You should avoid presenting the mathematical model immediately. Instead, facilitate a guided discovery process where learners analyze situations and gradually identify the decision variables, objective function, and constraints. This approach promotes conceptual understanding rather than procedural memorization.

Play a short video or presentation introducing Linear Programming (optional).

Ask learners to share key points they learned. You may list their responses on the board

Example 1. Recognizing LP Situations
Processing Questions: These questions guide learners to recognize the key components of Linear Programming (decisions, objective, and constraints), without immediately presenting the mathematical model.

Facilitate Example 2 to reinforce the identification of a Linear Programming situation after the farmer example. Ask learners to analyze the situation and identify what decisions must be made, what the goal is, and what limits the decisions. Guide learners to recognize that the bakery must decide how many brownies and cookies to produce, that the goal is to maximize profit, and that flour and time are constraints.

	• **Decision variables:** area planted to corn and area planted to beans • **Objective function:** maximize profit • **Constraint:** total land area is 10 hectares **Processing Questions** 1. To identify decision variables a. What choices does the farmer need to make? b. What quantities can change in this situation? c. What does the farmer decide about corn and beans? 2. To identify the objective (goal) a. What is the farmer trying to achieve? b. Is the farmer trying to increase something or decrease something? c. What does "maximize profit" mean in this context? 3. To identify constraints (limitations) a. What limits the farmer's decisions? b. How much land is available? c. Can the farmer use more than 10 hectares? Why or why not? 4. To check if it is a Linear Programming situation a. Are the quantities involved measurable? b. Is there a clear goal to optimize? c. Are there limits or restrictions on the choices? d. Can this situation be represented using numbers and inequalities? 5. To deepen understanding a. What would happen if there were no land limit? b. Why do you think land is an important constraint in this problem? **Example 2: Business Production** A bakery produces brownies and cookies. It aims to maximize profit but has limited flour and time. • **Decision variables:** number of brownies and number of cookies produced • **Objective function:** maximize profit • **Constraint:** limited flour and available time	

	Part II. LP or Not LP? **Instruction**: Determine whether each scenario can be modeled using LP. a. Planning meals with nutritional requirements (LP) b. Choosing clothes for a party (Not LP) c. Scheduling workers with limited working hours (LP) d. Allocating budget across school projects (LP) **Processing Questions** 1. "What quantities can be measured in this situation?" 2. "Is there a clear goal to maximize or minimize?" 3. "Are there restrictions or limitations involved?" **Mini-Practice: Anchor the Maximization Template** A school canteen sells turon (x pieces) and banana cue (y pieces) per day. Each turon earns ₱8 profit; each banana cue earns ₱6. The canteen has at most 30 oil-units per day; turon uses 2 oil-units, banana cue uses • Write the LP model in the four-part form (decision variables with units; objective; constraint; non-negativity) without solving it. **Processing Questions**: 1. Which variable letter did you use for the objective value? 2. Which inequality direction (≤ or ≥) did the phrase "at most 30 oil-units" produce? 3. Is your model a maximization or a minimization, and how does the direction word affect the optimum? **Part III. Linear Programming Model Construction** **1. Identify the Decision Variables** a. Decision variables are the unknown quantities that we want to determine. They represent the choices available in the problem. b. Each variable must represent a measurable quantity. c. Variables must be clearly and simply defined. d. They are usually represented by x, y (or more variables if needed), but the description of what they represent is more important than the symbol.	**Part II. LP or Not LP** 1. Present the situation. 2. Ask learners what quantities must be represented. 3. Assign variables based on learner suggestions. 4. Ask what the goal of the problem is. 5. Translate limitations into mathematical expressions. 6. Write each part on the board as students contribute. Learners explain why a scenario is or isn't an LP problem. **Mini-Practice: Anchor the Maximization Template.** **Big idea.** This mini-practice anchors the maximization template (capital P for the objective, ≤ for "at most" resource constraints) before the next example broadens scope. **Expected answers.** Decision variables: x = pieces of turon per day; y = pieces of banana cue per day. Objective: Maximize P = 8x + 6y, where P is daily profit in pesos. Constraint: 2x + y ≤ 30 (oil-units per day). Non-negativity: x ≥ 0, y ≥ 0. **Possible answers to process questions.** (1) P. (2) ≤. (3) Maximization; the direction word fixes whether the optimum is the corner farthest from the origin (max) or closest to it (min), with the same constraint set giving different optimal corners. **Anticipated misconception.** Students sometimes write the objective as "Income = 8x + 6y" without "Maximize" and without a symbol; redirect to the form "Maximize P = (coeff)x + (coeff)y." Materials: paper and pencil. No technology required. **Part III. Linear Programming Model Construction** **Probing Questions** 1. For identifying variables *"What quantities change in this situation?"* *"What are we trying to determine?"* 2. For writing the objective *"What outcome do we want to achieve?"* *"How does each variable contribute to that outcome?"* 3. For identifying constraints

	2. Write the Objective Function The objective function expresses what we want to **maximize** or **minimize**. It is based on the goal of the problem. There are two types: **a. Maximization** = e.g., profit, output, revenue, efficiency **b. Minimization** = e.g., cost, time, waste **How to write it:** 1. Multiply each decision variable by its contribution to the objective. 2. Add them together to create the objective function. **a. Write the Constraints** Constraints represent the **limitations** in the problem. *These may refer to:* ● Available resources (materials, time, labor, land, budget) ● Maximum production limits ● Storage, safety, or capacity restrictions ● Demand limits Constraints must be expressed as **linear inequalities or equations**, commonly: ● "At most" → ≤ ● "At least" → ≥ ● "No more than" → ≤ ● "No less than" → ≥ ● "Exactly" → = **How to write constraints:** 1. Identify the resource being limited. 2. Determine how much of that resource each product uses. 3. Combine them into an inequality. **4. Add the Non-Negativity Restrictions** In real life, we cannot produce negative amounts. Therefore, all variables must be restricted: **$x \geq 0$** **$y \geq 0$**	*"What resources are limited?"* *"How does each product use that resource?"* 4. Teaching Non-Negativity Restrictions Instead of stating the rule immediately, ask learners: *"Can the number of products produced be negative?"* *"Can the number of tools rented be negative?"* 5. Consolidating the Model a. Once all parts are constructed, show learners how to organize the information clearly. b. Structured Representation c. Demonstrate how the model is presented using four labeled sections: • Decision Variables • Objective Function • Constraints • Non-negativity Conditions/Restrictions Encourage learners to see the model as a systematic representation of the situation. 6. Deepening Understanding: Conceptual Question Ask the class the reflective question: *"Why is the method called Linear Programming?"* Allow learners to discuss in pairs before sharing answers. Guide the discussion toward the following ideas: *Why Linear?* • Because all equations and inequalities involve variables raised only to the first power, forming straight-line relationships. *Why Programming?* • Because the method involves planning and organizing decisions systematically to achieve the best result. Facilitate the second example of writing an LP model through guided practice.

	Key notes: ● This ensures the solution stays realistic. ● This condition is ALWAYS present in every LP problem. **5. Combine All Parts into a Full LP Model** After identifying variables, objective function, constraints, and non-negativity, we group them logically into one clear model. **Example 1. Baking Scenario** A baker wants to determine how many cupcakes and cookies to produce in order to maximize profit. Each cupcake yields a profit of ₱15, while each cookie yields a profit of ₱10. The baker has at most 300 minutes available for production. Producing one cupcake requires 5 minutes, and producing one cookie requires 3 minutes. In addition, the bakery can sell no more than 40 cupcakes. **1. Identify the Decision Variables** **Example:** A bakery makes cupcakes and cookies. Let: x = number of cupcakes to produce y = number of cookies to produce **2. Write the Objective Function** **Example:** Profit = ₱15 per cupcake, ₱10 per cookie Objective: **Maximize P = 15x + 10y** **3. Write the Constraints** a. Total baking time available: 300 minutes Cupcake uses 5 minutes; cookie uses 3 minutes Time Constraint: **5x + 3y ≤ 300** b. The bakery can sell no more than 40 cupcakes Demand Constraint: **x ≤ 40**	

4. Add the Non-Negativity Restrictions
$x \geq 0, y \geq 0$

5. Combine All Parts into a Full LP Model

LP Model:
Decision Variables: x = cupcakes, y = cookies
Objective Function: Maximize P = 15x + 10y, where P is total profit in pesos
Constraints:

- **$5x + 3y \leq 300$**
- **$x \leq 40$**
- **$x, y \geq 0$**

Example 2. Repair Shop
A repair shop needs to minimize its weekly tool rental cost. It rents soldering irons at ₱120 each, with each unit handling 5 repair jobs per week, and multimeters at ₱180 each, with each unit handling 3 repair jobs per week. The shop must complete at least 30 repair jobs weekly, rent at least 4 multimeters for complex work, and can rent no more than 8 soldering irons due to limited supply. Formulate a linear programming model to determine how many of each tool should be rented

1. Decision Variables
Let *I = number of soldering irons rented per week*
M = number of multimeters rented per week

Objective Function (Minimization). Minimize C = 120I + 180M, where C is the total weekly rental cost in pesos.
Constraints. Job-capacity requirement: 5I + 3M ≥ 30. Minimum multimeters for diagnostics: M ≥ 4. Maximum available soldering irons: I ≤ 8.
Non-negativity. I ≥ 0, M ≥ 0.
LP Model. Decision variables: I = number of soldering irons rented per week; M = number of multimeters rented per

<table>
<tr><td></td><td colspan="2">week. Objective function: Minimize C = 120I + 180M. Constraints: 5I + 3M ≥ 30; M ≥ 4; I ≤ 8. Non-negativity: I ≥ 0, M ≥ 0.

Question: “Why is it called Linear Programming? Why linear? Why programming?”</td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery</td></tr>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS
1. Divide the class into groups.
2. Distribute the learning activity sheet.
3. Ask learners to carefully read the instructions and accomplish the tasks accurately.
4. During group discussions, move around the classroom and observe how learners analyze the models.
5. When groups struggle, guide them using probing questions rather than providing direct answers.
Examples of guiding questions include:
a. Does the objective function reflect the goal stated in the problem?
b. Do the constraints represent the limitations mentioned in the scenario?
c. Are the inequality symbols consistent with the meaning of the statements?
d. Do the variables represent the correct quantities in the situation?
6. Encourage learners to revisit the original scenario whenever they are unsure about the correctness of a statement in the model.
7. After the activity, let the learners reflect on their own learning experience by the answering the following questions individually:
a. What types of errors did you notice most often in the flawed LP models, and how did you identify them?
b. How did correcting the models help deepen your understanding of how LP problems are properly formulated?
c. Which part of the LP model (variables, objective function, constraints, or non-negativity) do you still find challenging to analyze or correct? Why?</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
1. Create a real-world problem that involves decision-making under constraints. You may choose from the following themes:

❄️ School event (fundraising, booths, food sale)
🧁 Small business (pastries, crafts, printing services)
🚜 Agriculture (crop planning, livestock feeding mix)
🏀 Sports scheduling (training time allocation)</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

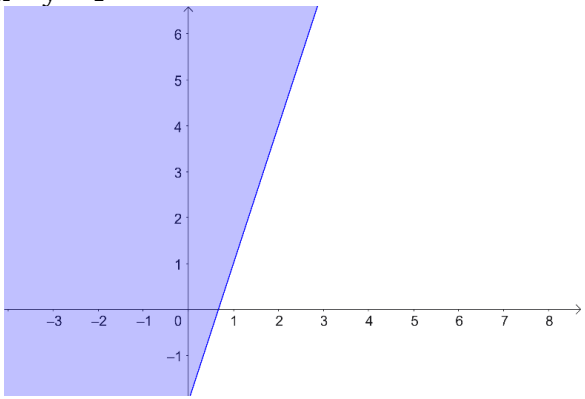
1. Introduce the Task
2. Present the themes.
3. Give learners instructions
a. Choose one theme.
b. Create a short real-life situation with:
• Two things to decide
• An objective (maximize or minimize)
• At least two numerical constraints
4. Translate the situation into an LP model with:
a. Decision variables</td></tr>
</table>

	🏠 **Home/Personal** (study hours, budget allocation, meal planning) 🚚 **Community** (relief goods distribution, barangay planning) 2. Translate the situation into a complete Linear Programming (LP) model: a. Decision variables b. Objective function c. Constraints d. Non-negativity	b. Objective function c. Constraints d. Non-negativity 5. After completing the task of creating and translating a real-world Linear Programming problem, facilitates a reflective activity to help learners assess their learning. a. What part of creating your own real-world Linear Programming problem did you find most challenging, and how did you address it? b. How did translating a real-life situation into decision variables, an objective function, and constraints help you better understand Linear Programming? c. Which aspect of the LP model do you still find confusing or need more practice with, and why? 6. Call some learners to share their responses to encourage reflection and discussion.
	C.2. Making Generalization	
	Activity C.2. WRAPPING UP THE LESSON **Option 1. Sentence Completion** Fill in the blanks based on what you learned in the lesson. 1. A Linear Programming problem always involves deciding between _____. 2. The goal of Linear Programming is to maximize _____or minimize _____. 3. Constraints represent limits on _____. 4. Variables must be non-negative because _____. **Option 2. 3-2-1 Ladder Reflection** To end the lesson write: 3 things all Linear Programming models have 2 reasons Linear Programming is helpful 1 example of where they could apply Linear Programming in real life **Option 3. One-Minute Model Builder** Answer the prompt in one minute: “In one sentence, describe what Linear Programming is and when it is used.”	**Activity C.2. WRAPPING UP THE LESSON** To synthesize the lesson, choose one of the provided activities. Have learners share their responses with the class and facilitate a discussion to process their insights and learning.

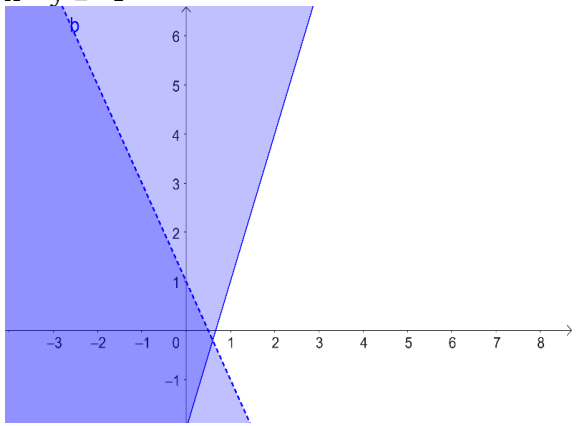
	C.3. Evaluating Learning
	Activity C.3. ASSESSING LEARNING OUTCOMES Before facilitating this activity, ensure that all learners have a copy of the LAS 1. To assess learners' understanding of the lesson, facilitate the evaluation activity. After reviewing their responses, guide learners in a brief reflection to evaluate their learning and identify areas for further improvement. 2. Invite selected learners to share their responses to encourage reflection and discussion. Learners' answers are used as formative assessment to identify misconceptions and guide remediation or enrichment activities. a. How has your understanding of decision-making changed after learning LP, especially in situations with limited resources? b. Which concepts or skills in Linear Programming did you find easiest or most difficult, and why? c. What aspects of Linear Programming do you still find confusing or need more practice with?
	C.4. Additional Activities
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS Based on the results of the evaluation, facilitate the appropriate learning activities. Learners who have not yet attained mastery should complete the remediation activity, while those who have attained mastery should be provided with the enhancement activity. During the remediation activity, teacher support is essential. Provide timely guidance, feedback, and scaffolding to help learners address misconceptions, strengthen their understanding of the concepts, and progress toward mastery.

III. CONTENT	**Lesson 4. 2. GRAPHICAL METHOD FOR SOLVING LINEAR PROGRAMMING PROBLEMS**	
IV. OBJECTIVES	**By the end of the lesson, learners are able to:** **1. identify the components of linear programming such as the decision variables, objective function, constraints, non-negativity restrictions, and feasible region;** **2. illustrate the feasible region of a Linear Programming (LP) problem;** **3. identify the corner points (vertices) of the feasible region;** **4. determine the optimal solutions to a LP problem using the Corner Point Theorem; and** **5. solve linear programming problems of two variables by graphical method, with or without technology.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1.* Eliciting *Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS Instruction: Graph the following systems of linear inequalities and identify the shaded region that represents the solution of such inequalities.	**Activity A.1. LEVELING LEARNER READINESS** Recall the concept of linear inequalities by presenting two examples. Then, facilitate a review of the steps in solving linear inequalities graphically as learners discuss the answers to the guide questions.

1. $3x - y \leq 2$
   $2x + y < 1$



2. $x + 2y \leq 6$
   $x - y \geq -1$



Process Questions:

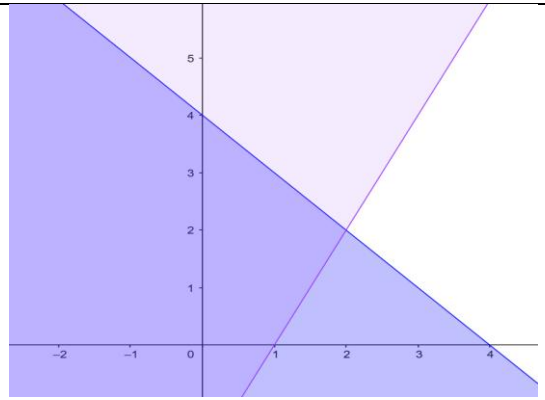
1. How are linear inequalities in two variables solved graphically?
2. How do you determine whether an ordered pair is a solution to a given linear inequality or a system of linear inequalities?
3. What do you notice about the shaded region when two inequalities overlap, and which part of that overlap represents all possible solutions to the system?

Emphasize that the solution to a system of linear inequalities is the region where the solution sets of the inequalities overlap. Clarify that if the boundary line is dashed (broken), the points on the line are not included in the solution set; if the boundary line is solid, the points on the line are included in the solution set.

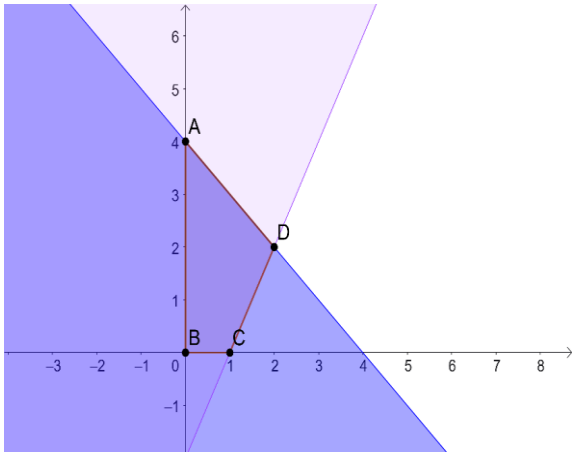
To explain the answers to the activity, let the learners share their thoughts on the importance of graphing systems of linear inequalities. Accept varied responses from the learners.

	A.2. Establishing the Purpose of the Lesson	
	Activity A. 2. APPRECIATING LESSON RELEVANCE **Instructions**: Read and solve the problem below, then answer the processing questions. During a school fair, a class plans to sell burgers and hotdogs to raise funds. Each burger yields a profit of ₱10.00, while each hotdog yields a profit of ₱5.00. However, the following constraints must be considered: 1. The booth can prepare at most 80 food items per day. 2. Due to limited ingredients, at most 40 burgers can be prepared. 3. The number of hotdogs must be at most 50 to meet customer demand. **Process Questions:** 1. What are the decision variables in the problem? 2. What is the objective function in the problem? 3. What are the limitations or constraints given in the problem? 4. How can these constraints be expressed as mathematical inequalities? 5. If these inequalities are graphed, what points satisfies all the conditions?	**Activity A. 2. APPRECIATING LESSON RELEVANCE** Display the burger / hotdog scenario, then use the process questions to lead learners through naming the decision variables, objective, and constraints in their own words. Do not write the formal LP model on the board yet; the goal is recognition, not formalization. Accept informal phrasing such as 'how many burgers and how many hotdogs' for decision variables. **Misconception watch.** Some learners read 'at most 50 hotdogs' as 'exactly 50 hotdogs'. Clarify that 'at most' means '≤ 50, including any value from 0 to 50'.
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Corner Point Discovery** **Instructions**: Below are three polygons drawn on graph paper. Without using a formula, mark every point where two boundary lines meet. Compare your marks with a partner. List the marked points as ordered pairs (x, y). **Processing questions**:	**Activity B.1. PRESENTING EXAMPLES** **Corner Point Discovery** Present the Fundamental Theorem of Linear Programming and explain how the corner points of the feasible region are used to determine the minimum and maximum values of the objective function. **Explain:** In linear programming, we aim to find the best possible value (maximum profit or minimum cost) subject to

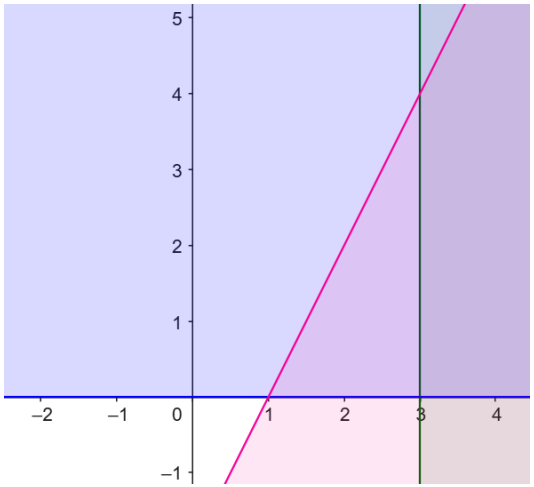
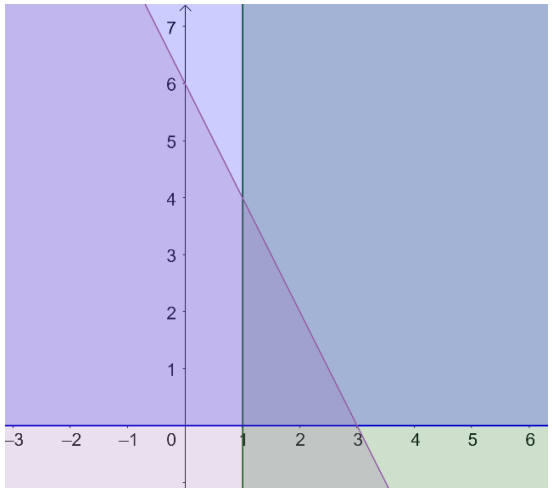
	1. What did every marked point have in common? 2. Could a point inside the polygon (not on any boundary) be marked? Why or why not? 3. If you were asked to find the point in the polygon that maximizes a linear quantity such as 4x + 3y, which marked points would you check first? **Fundamental Theorem of Linear Programming (Corner Point Theorem)** "If an optimal solution exists, it occurs at one of the corner points (vertices) of the feasible region. This means that instead of testing all points in the feasible region, we only evaluate the objective function at the corner points to determine the optimal solution." **Graphical Method for Solving Linear Programming Problems** "The graphical method is used to solve linear programming problems involving two decision variables only. To formulate a linear programming problem, we define the decision variables, objective function, constraints, and non-negativity restrictions." Define **x, y,** and ***z*** Maximize or minimize $z = ax + by$ Subject to (Constraints) $cx + dy \leq c_1$ $ex + fy \leq c_2$ Non-negativity restrictions $x, y \geq 0$ Graph the constraints	certain limitations called constraints. When these constraints are graphed on a coordinate plane, they form a region called the feasible region. Based on the Fundamental Theorem of Linear Programming, the graphical method becomes an effective way to solve linear programming problems. By graphing the constraints, identifying the feasible region, and evaluating the corner points, we can determine the maximum or minimum value of the objective function. **Note:** x represents the first quantity y represents the second quantity **Misconception watch.** Learners may list only the vertices that lie on the boundary lines they drew last and miss the intersection of two earlier boundary lines. *Prompt: 'Walk around the polygon and mark each corner where two lines meet.'*

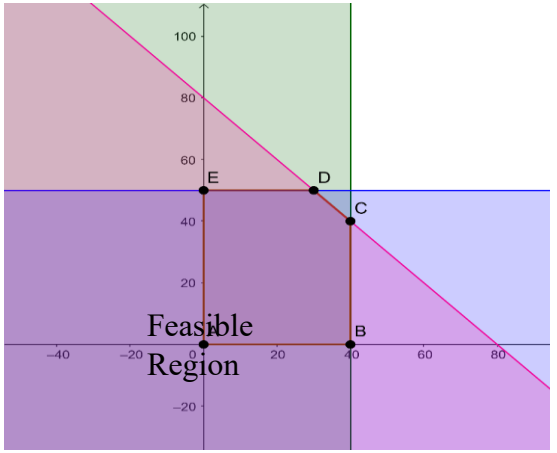


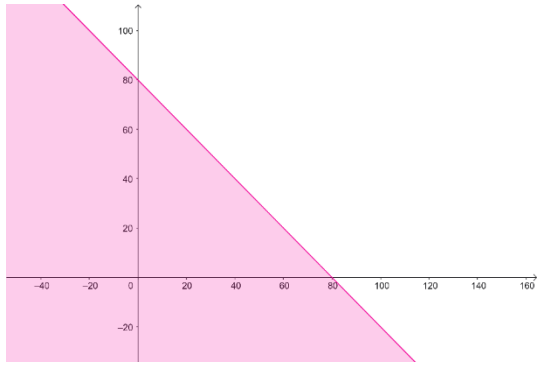
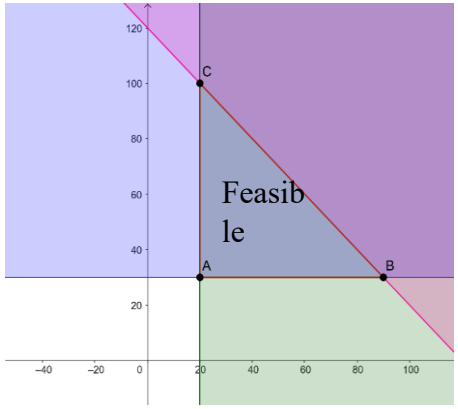
The ***feasible region*** is the solution set of the system of linear inequalities derived from the given constraints. It forms a polygon on the coordinate plane. The corner points of this region are the vertices of the polygon.

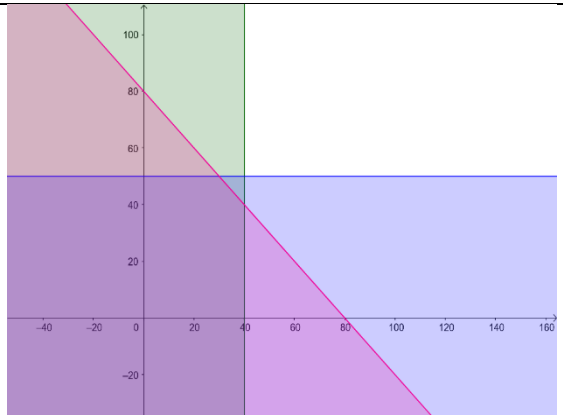


Feasible region

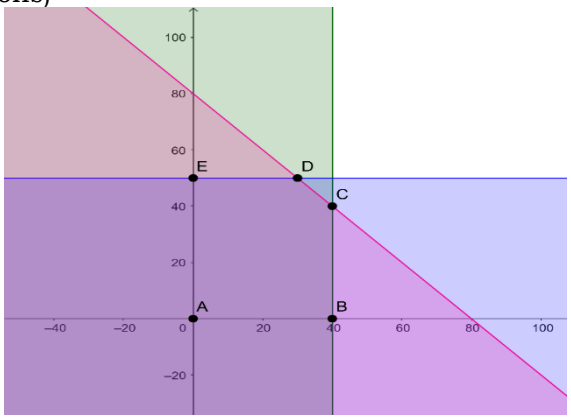
	Identify the corner points of the given graphs. 1.  2.  	
	B.2. Discussing the Concept	
	Activity B.2. DEPEENING CONCEPTUAL UNDERSTANDING **Instructions**:	**Activity B.2. DEPEENING CONCEPTUAL UNDERSTANDING**

	The problems below are examples of how to solve linear programming problems. To fully master today's lesson, please listen attentively and actively participate in the discussion. **Discussing the Graphical Method Using the Problem** Example 1. During a school fair, a class plans to sell burgers and hotdogs to raise funds. Each burger yields a profit of ₱10.00, while each hotdog yields a profit of ₱5.00. However, the following constraints must be considered: 1. The booth can prepare at most 80 food items per day. 2. Due to limited ingredients, at most 40 burgers can be prepared. 3. The number of hotdogs must be at most 50 to meet customer demand. **Solutions**: Step 1. Decision Variables (These represent the quantities to be determined.) • Let x be the number of burgers to be sold. • Let y be the number of hotdogs to be sold. Step 2: Objective Function (This represents the goal of the problem, usually maximizing profit.) Each burger gives a profit of ₱10, and each hotdog gives a profit of ₱5. Maximize: $z = 10x + 5y$ where z is the total profit. Step 3: Constraints (These are the limitations given in the problem.) a. The booth can prepare at most 80 food items per day, which can be expressed as: $x + y \leq 80$ b. Due to limited ingredients, at most 40 burgers can be prepared. $x \leq 40$	The problem serves as a springboard for discussing the steps in translating constraints into linear inequalities and representing them graphically. Through guided learning, learners identify the feasible region and understand how mathematical models are formed. Explain that the region bounded by the corner points is called the feasible region. Reiterate that the ***feasible region*** is the solution set of the system of linear inequalities derived from the given constraints. It forms a polygon on the coordinate plane. The corner points of this region are the vertices of the polygon  Decision Variables: • Let x be the liters of gasoline • Let y be the liters of diesel Objective Function: $z = 62x + 57y$ Constraints: $\begin{cases} x + y \geq 120 \\ x \geq 20 \\ y \geq 30 \\ x, y \geq 0 \end{cases}$

	The number of hotdogs must be at most 50 to meet customer demand: y ≤ 50. Non-negative constraints $x \geq 0, y \geq 0$ Step 4: Graph the Constraints Convert each into boundary lines. Graphing $x + y \leq 80$ $y \leq 50$  	Graph:  Discuss to the learners that the values of x and y were substituted to the objective function to evaluate if the solutions are optimal. **Technology integration (optional).** Demonstrate the burger hotdog example in GeoGebra (Tools > Inequality) to show the feasible region forming as each inequality is added. Drag a point inside the region and watch z = 10x + 5y update; learners observe the maximum sits at a corner. Non-tech alternative: present the same example on a printed coordinate plane, shading each half-plane with a different color so the overlap is visible.



Step 5: Corner Points of the Feasible Region (Find intersections)



Point of intersections.

A(0,0), B(40,0), C(40,40), D(30,50), E(0,50)

Step 6: Evaluate the Objective Function

$$z = 10x + 5y$$

Corner	(x,y)	Profit
A	(0,0)	10(0) + 5(0) = 0 + 0 = 0
B	(40,0)	10(40) + 5(0) = 400 + 0
C	(40,40)	10(40) + 5(40) = 400 + 200 = 600

D	(30,50)	10(30) + 5(50) = 300 + 250 = 550	
E	(0,50)	10(0) + 5(50) = 0 + 250 = 250	

Step 7: Optimal Solution
The maximum profit is 600 at (x,y) = (40, 40). This means that selling 40 burgers and 40 hotdogs yields the highest profit while satisfying all constraints. This solution remains valid since it also satisfies the condition for hotdogs which is $y \leq 50$, since $40 \leq 50$.

Even with the restriction of at most 50 hotdogs, the optimal solution is still 40 burgers and 40 hotdogs, with a total profit of ₱600.

Processing Questions:

1. Did the shaded region for the graphed linear inequalities overlap?
2. What points bounded the region where graphed linear inequalities overlapped?

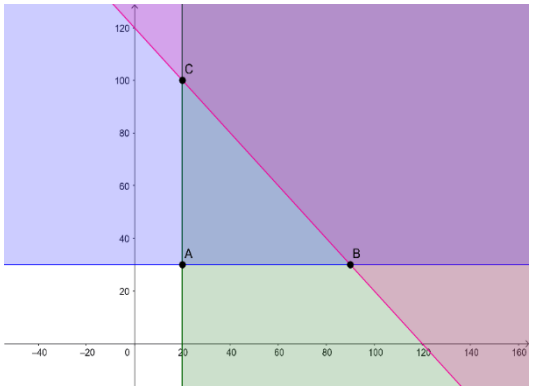
Example 2.
A delivery company needs at least 120 liters of fuel per day for its vehicles. The company can purchase gasoline or diesel from a nearby gas station. Gasoline costs ₱62 per liter, while diesel costs ₱57 per liter.

Operational Requirements:

1. The company must purchase at least 20 liters of gasoline for motorcycles.
2. The company must purchase at least 30 liters of diesel for delivery vans.
3. The manager aims to determine the number of liters of gasoline and diesel to purchase in order to minimize the total fuel cost

Process Questions:

1. What are the decision variables in the problem?
2. What is the objective function of the problem?
3. What are the constraints given in the problem?
4. What are the corner points and the feasible region?



Candidate point A(20, 30) is the intersection of the boundaries x = 20 and y = 30 and is therefore generated by the lower-bound constraints alone. It is not feasible because x + y = 20 + 30 = 50, which is less than the required minimum of 120 liters; the constraint x + y ≥ 120 fails. Do not list A(20, 30) as a corner of the feasible region. The feasible corner points are B(90, 30), from y = 30 and x + y = 120, and C(20, 100), from x = 20 and x + y = 120.

5. What is the optimal solution?

$$z = 62x + 57y$$

Corner	(x,y)	Total cost
A	(20,30)	Not feasible
B	(90,30)	62(90) + 57(30) = 5580 +1710 = 7290
C	(20,100)	62(20) + 57(100) = 1240 + 5700 = 6940

Optimal Solution
The minimum cost is ₱6,940, which occurs at (20, 100). This means that the company should purchase 20 liters of gasoline and 100 liters of diesel to achieve the minimum total fuel cost of ₱6,940.

Direction: Find the maximum and the minimum values of each given objective function subject to the indicated constraints.

	1. $z = 3x + 2y$ $\begin{cases} x - y \leq 1 \\ x + 2y \leq 4 \\ x, y \geq 0 \end{cases}$ 2. $z = 3x + 4y$ $\begin{cases} x + 2y \leq 10 \\ 3x - y \geq 0 \\ x - y \leq 2 \\ x, y \geq 0 \end{cases}$ 3. $p = 30x + 40y$ $\begin{cases} 2x + 4y \leq 120 \\ x + y \leq 50 \\ x, y \geq 0 \end{cases}$ 4. $s = 8x + 6y$ $\begin{cases} x + y \leq 10 \\ x \geq 50 \\ y \leq 6 \\ x, y \geq 0 \end{cases}$	
	B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)	
	Activity B.3. PRACTICING LEARNED SKILLS Before facilitating this activity, ensure that all learners have a copy of the LAS. Let learners practice the full graphical method on two contrasting problems: a minimization with a mixed-direction constraint (Problem 1) and a maximization with three resource constraints (Problem 2). Guide learners to apply every step from B.2 independently. Facilitate the facilitate the processing questions afterwards **Misconception watch.** Some learners list all axis-intercepts of every constraint line as corner points, including those outside the feasible region. Prompt: 'A corner point qualifies only if it satisfies every constraint. Substitute to check.'	
C. Demonstrating Knowledge and Skills	***C.1. Finding Practical Application***	
	Activity C.1. MAKING REAL-WORLD CONNECTIONS **Instruction:** Solve the linear programming problem and show the solutions in front of the class. **Situation 1.** A grade 11 student is starting a small snack business selling banana cue and camote cue. Each stick of banana cue earns ₱10 profit and each stick of camote cue earns ₱8 profit However, there are daily resource constraints	**Activity C.1. MAKING REAL-WORLD CONNECTIONS** **Big idea.** C.1 is the first time learners apply the full graphical method to a real-world context they choose. The two situations differ only in coefficients and constraints, so the method is identical; the contrast helps learners see that the procedure generalizes. **Facilitation.** Divide the class into groups of three or four. Assign or let each group choose one of the two situations. Allocate 30 minutes for the group to (a) identify decision variables, objective function, and constraints; (b) graph the

a. The student only has enough ingredients to make at most 40 sticks total
b. Preparing banana cue takes more time, so she can only make at most 25 sticks of banana cue per day
c. She must make at least 10 sticks of camote cue to meet demand.

Situation 2: A student-farmer is managing a small farm where they can plant corn and vegetables. Each hectare of corn gives a profit of ₱ 5 000 and each hectare of vegetables gives a profit of ₱ 4 000.
However, the farm has limited resources:
a, The total land available is 10 hectares
b. Corn requires more water, so only 6 hectares can be used for corn
c. At least 2 hectares must be planted with vegetables to meet market demand.

feasible region; (c) evaluate the objective at every corner point; (d) state the optimum in context. Allocate 15 minutes for one representative per group to present and explain; collect work for assessment.

Misconception watch. Some groups will state the optimum as '(25, 15)' without units or interpretation. Prompt: 'State your answer as a complete sentence in the language of the scenario: how many of each product, and what is the profit?'

Processing Questions:
1. Why is your solution the most efficient and profitable among all possible combinations, and what would happen if one constraint changed?
2. How can the process you used in solving this problem be applied to a real-life decision-making situation involving limited resources

C.2. Making Generalization

Activity C.2. WRAPPING UP THE LESSON

Instructions:
Complete the organizer below to summarize the key concepts learned in solving linear programming problems using the graphical method.

Components	Questions	Responses
Real-life situation	What practical situations can be modeled using linear programming?	
Decision Variables	What quantities are represented by the variables?	
Objective Function	What are we trying to maximize or minimize?	
Constraints	What limitations or conditions must be satisfied?	

Activity C.2. WRAPPING UP THE LESSON

To synthesize the lesson, allow learners to complete the organizer individual, then pair learners to compare responses. Use the remaining time to address the four Processing Questions.

Feasible Region	What does the feasible region represent in the graph?	
Corner Points	Why do we evaluate the objective function on the corner points?	
Optimal Solution	What solution gives the best possible result?	
Real-life Meaning	How does the solution help in making decisions in real life?	

Process Questions:

1. What is the importance of identifying constraints in real-life problems?
2. Why do we focus on the corner points of the feasible region when solving linear programming problems?
3. How can linear programming help people make better decisions in everyday life?
4. What happens if there is no feasible region?

C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)

Activity C.3. ASSESSING LEARNING OUTCOMES

Before facilitating this activity, ensure that all learners have a copy of the LAS

1. To assess learners' understanding of the lesson, facilitate the evaluation activity. After reviewing their responses, guide learners in a brief reflection to evaluate their learning and identify areas for further improvement.
2. Invite selected learners to share their responses to encourage reflection and discussion. Learners' answers are used as formative assessment to identify misconceptions and guide remediation or enrichment activities.
   a. How has your understanding of decision-making changed after learning LP, especially in situations with limited resources?
   b. Which concepts or skills in LP did you find easiest or most difficult, and why?
   c. What aspects of LP do you still find confusing or need more practice with?

C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)

Activity C.4: Extending and Reinforcing Learning.

Before facilitating this activity, ensure that all learners have a copy of the LAS

Based on the results of the evaluation, facilitate the appropriate learning activities. Learners who have not yet attained mastery should complete the remediation activity, while those who have attained mastery should be provided with the enhancement activity.

	During the remediation activity, teacher support is essential. Provide timely guidance, feedback, and scaffolding to help learners address misconceptions, strengthen their understanding of the concepts, and progress toward mastery.

III. CONTENT	**Lesson 4.3. SOLVING LINEAR PROGRAMMING PROBLEMS USING SIMPLEX METHOD**	
IV. OBJECTIVES	**By the end of the lesson, learners are expected to:** **1. identify and set up a linear program in standard maximization form;** **2. convert inequality constraints to equations using slack variables;** **3. set up the initial simplex tableau by performing pivoting operations; and** **4. identify the optimal solution from the optimal simplex tableau.**	
V. PROCEDURES	**LEARNERS ACTIVITIES**	**ANNOTATION (FACILITATION GUIDE)**
A. Activating Prior Knowledge	***A.1. Eliciting Prior Knowledge***	
	Activity A.1. LEVELING LEARNER READINESS Instruction: Given maximize function $P = 30x + 40y$, maximize subject to the following constraints: $$\begin{cases} 2x + y \leq 10 \\ x + y \leq 7 \\ x + 2y \leq 12 \\ x, y \geq 0 \end{cases}$$ **Processing Questions:** 1. What are the corner points of the feasible region in this problem? 2. Why do we need to evaluate the objective function? 3. What is the optimal value? 4. What is your conclusion?	**Activity A.1. LEVELING LEARNER READINESS** **Facilitation.** Give learners 8 minutes to graph the constraints and identify the five corners. Walk the room and check that learners label corners A through E. Then 6 minutes to evaluate P at each corner and identify the maximum. Close with 6 minutes on the four process questions, taken orally as a whole class. is 260 pesos. (4) The producer should choose x = 2 and y = 5 to maximize profit at P = 260 pesos. **Bridge to B.1.** Tell learners: "You just solved this LP graphically. In B.1 we will solve the same LP again by the simplex method, and you will get the same answer." This sets up the cognitive bridge that LE16 needs.
	A.2. Establishing the Purpose of the Lesson	
	Activity A.2. APPRECIATING LESSON RELEVANCE **Instructions**: The following essential questions will help you realize the relevance of today's lesson. By answering these, you can engage more deeply in our discussion. 1. How can we determine the best possible solution when there are multiple constraints?	**Activity A. 2: Appreciating Lesson Relevance** 1. Introduce the lesson and discuss its importance using essential questions. 2. Call some learners to share their answers. 3. Introduce the objectives of the lesson.

	2. Why is it important to optimize resources in real-life situations? 3. How does the simplex method improve decision-making compared to graphing methods? 4. What does an "optimal solution" mean and how do we reach it?	
B. Instituting New Knowledge	***B.1. Presenting Examples***	
	Activity B.1. EXPLORING KEY CONCEPTS **Instructions:** To show you how to solve a real-life linear programming problem, let's use a classic business scenario: **maximizing profit under resource constraints**. **Situation:** A small bakery produces loaves of bread and cakes. Each loaf earns a profit of Php30, while each cake earns Php40. The bakery wants to maximize its total profit. However, production is limited by the following resources: 1. Each loaf requires 2 hours of labor and each cake requires 1 hour. The company has at most 10 labor hours available per day. 2. Each loaf and cake both use 1 unit of raw material, and only 7 units are available. 3. Each loaf uses 1 unit of another material, while each cake uses 2 units. There only 12 units available. **Step 1. Set up the problem.** Write the objective function and the constraints. Since the simplex method is used for problems that consists many variables, we can use symbols x_1, x_2, x_3, and so on. Let x_1 = loaves of bread x_2= pieces of cakes The problem is formulated the same way as we did for graphical methods. Maximize: $Z = 30x_1 + 40x_2$ Subject to:	**Activity B.1. EXPLORING KEY CONCEPTS** Guide the learners to translate the real-life situation into linear programming model. The problem is a linear programming maximization problem with three inequality constraints and two variables. To solve it using the Simplex method, we first convert inequalities into equalities by adding slack variables. Then, we set up the initial simplex tableau and perform pivot operations to optimize the objective function. In Step 1, after writing the objective function and constraints, decision variables are represented as $x_1, x_2, x_3 \ldots$ to prepare students for the simplex method. Although earlier sections may use x and y, explain that these are simply renamed as $x_1$and x_2 . The change is intentional and supports the simplex method's ability to handle multiple variables systematically. **(Standard form)** Inequalities are converted into equalities by adding non-negative slack or surplus variables. **(Tableau Method)** Calculations are typically organized in a "simplex tableau" to perform row operations and check for optimality. The objective-row entries are written as the negatives of the original objective coefficients because we rearrange the objective Z = c1 x1 + c2 x2 + … as Z - c1 x1 - c2 x2 - … = 0 before putting it in the tableau. This is bookkeeping, not a max-to-min conversion: the simplex method is solving the original maximization problem directly.

$$\begin{cases} 2x_1 + x_2 \le 10 \\ x_1 + x_2 \le 7 \\ x_1 + 2x_2 \le 12 \\ x_1, x_2 \ge 0 \end{cases}$$

Step 2. Introduce slack variables.
Add slack variables s_1, s_2, and s_3 (one per ≤ constraint) to convert the three inequalities into equalities. Re-write the objective function with all terms on one side: $Z - 30x_1 - 40x_2 = 0$, which gives the Z-row entries -30 (under x1), -40 (under x_2), and 0 (under each of s_1, s_2, s_3).
Standard form:
$2x_1 + x_2 + s_1 = 10$
$x_1 + x_2 + s_2 = 7$
$x_1 + 2x_2 + s_3 = 12$
$Z - 30x_1 - 40x_2 = 0$
Non-negativity: $x_1, x_2, s_1, s_2, s_3 \ge 0$.

Step 3:Set up the initial simplex tableau

Basis	x_1	x_2	s_1	s_2	s_3	Z	RHS
s_1	2	1	1	0	0	0	10
s_2	1	1	0	1	0	0	7
s_3	1	2	0	0	1	0	12
Z	-30	-40	0	0	0	1	0

Step 4. Identify entering variable
Look for the most negative coefficient in the Z-row (objective function row):

- Coefficients: -30 (x_1), -40 (x_2),
- Most negative is -40 corresponding to x_2

So, x_2 will enter the basis.

Pause and predict. Before doing any row operation, answer the following questions in your notebook.
1. Which variable should enter the basis first? Explain using the Z-row.

RHS – right hand side

Basis	x_1	x_2	s_1	s_2	s_3	Z	RHS
s_1	2	1	1	0	0	0	10
s_2	1	1	0	1	0	0	7
s_3	1	2	0	0	1	0	12
z	-30	-40	0	0	0	1	0

$R_3 \div 2$

s_3 becomes x_2 on the new tableau.

$R_1: R_1 - R_3$

$R_2: R_2 - R_3$

$R_3: 40R_3 + R_4$

Note: Iterative Process - Starts at an initial basic feasible solution and moves to an adjacent vertex to increase/decrease the objective function value.)

Basis	x_1	x_2	s_1	s_2	s_3	Z	RHS
s_1	$\frac{3}{2}$	0	1	0	$-\frac{1}{2}$	0	4
s_2	$\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	0	1
x_2	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	6
z	-10	0	0	0	20	1	240

$R_2: R_2(2)$

$R_1: R_1 - \frac{3}{2}R_2$

$R_3: R_3 - \frac{1}{2}R_2$

2. Which slack variable should leave the basis? Show the minimum-ratio test.
3. What does the leaving variable tell us about the limiting resource?

Step 5. Identify leaving variable
Calculate the minimum ratio of RHS to the positive coefficients in the column:

- For s_1: $\frac{10}{1} = 10$
- For s_2: $\frac{7}{1} = 7$
- For s_3: $\frac{12}{2} = 6$

Minimum ratio is 6, so s_3 will leave the basis.

Step 6. Pivot on element in row s_3, column x_2
Make the pivot element 1 by dividing row s_3 by 2:

Basis	x_1	x_2	s_1	s_2	s_3	Z	RHS
s_1	2	1	1	0	0	0	10
s_2	1	1	0	1	0	0	7
x_2	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	6
z	-30	-40	0	0	0	1	0

Step 7. Make other entries in x_2 column zero

Basis	x_1	x_2	s_1	s_2	s_3	Z	RHS
s_1	$\frac{3}{2}$	0	1	0	$-\frac{1}{2}$	0	4
s_2	$\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	0	1
x_2	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	6
z	-10	0	0	0	20	1	240

Step 8. Repeat - Identify the entering variable.

R_4: $10R_2 + R_4$

Optimality Test. The current BF solution is optimal if and only if every coefficient in row 0 is nonnegative (≥ 0).

	Look for the most negative coefficient in the Z-row: the only negative entry is -10 (under x_1). So x_1 enters the basis. **Step 9. Identify the leaving variable.** Compute the minimum ratio of RHS to the positive entries in the x_1 column: For s_1 row: $4 \div (3/2) = 8/3 \approx 2.67$ For s_2 row: $1 \div (1/2) = 2$ For x_2 row: $6 \div (1/2) = 12$ Minimum ratio is 2 (s2 row), so s2 leaves the basis. **Step 10. Pivot on the (s2 row, x1 column) entry.** Divide the s2 row by the pivot value 1/2 to make the pivot equal to 1; the s2 row becomes the new x1 row: [1, 0, 0, 2, -1, 0, 2]. **Step 11. Make the other entries in the x_1 column zero, then read off the solution.** After elimination, the final tableau is: s_1: [0, 0, 1, -3, 1, 0, 1] x_1: [1, 0, 0, 2, -1, 0, 2] x_2: [0, 1, 0, -1, 1, 0, 5] Z: [0, 0, 0, 20, 10, 1, 260] All Z-row coefficients are non-negative, so the solution is optimal. Basic variables: $x_1 = 2$, $x_2 = 5$, $s_1 = 1$. Non-basic: $s_2 = s_3 = 0$. Maximum value of Z (profit P) = 260. **Conclusion.** The bakery should produce 2 loaves of bread and 5 pieces of cakes to make a maximum profit of ₱260.	
	B.2. Discussing the Concept	
	Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING **Instructions:** The problems are examples of solving LP using simplex method. To understand the process listen attentively and participate actively during discussion. **Scope and Conditions of the Simplex Method**	**Activity B.2. DEEPENING CONCEPTUAL UNDERSTANDING** 1. Begin the discussion by reviewing the conditions required for applying the Simplex Method. Ask learners to examine the given linear programming problem and verify that it satisfies all the conditions for the method. Emphasize the role of the objective function, constraints, non-negativity restrictions, and slack variables.

1. The LP must be a MAXIMIZATION problem (we maximize $Z = c_1 x_1 + c_2 x_2 + \ldots + c_k x_k$).
2. Every structural constraint must be of the form $a x_1 + b x_2 + \ldots \leq r$, with the right-hand side r non-negative.
3. Every decision variable must satisfy non-negativity: $x_1, x_2, \ldots, x_k \geq 0$.
4. There is one slack variable per ≤ constraint, so the slack variables form the initial basis with values equal to the RHS of each constraint.

Solve using the Simplex method.
Maximize:
$Z = 10x_1 + 15x_2 + 20x_3$
Subject to constraints:
$2x_1 + 4x_2 + 6x_3 \leq 24$
$3x_1 + 9x_2 + 6x_3 \leq 30$
$x_1, x_2, x_3 \geq 0$

Step 1. Set up the problem.
Maximize:
$Z = 10x_1 + 15x_2 + 20x_3$
Subject to:
$2x_1 + 4x_2 + 6x_3 \leq 24$
$3x_1 + 9x_2 + 6x_3 \leq 30$
$x_1, x_2, x_3 \geq 0$

Step 2. Introduce slack variables. Add slack variables s_1 and s_2 to convert the ≤ constraints into equations.
Standard form:
$2x_1 + 4x_2 + 6x_3 + s_1 = 24$
$3x_1 + 9x_2 + 6x_3 + s_2 = 30$
$Z - 10x_1 - 15x_2 - 20x_3 = 0$
$x_1, x_2, x_3, s_1, s_2 \geq 0$

Step 3. Set up the initial simplex tableau.

Basis	x_1	x_2	x_3	s_1	s_2	RHS
s_1	2	4	6	1	0	24
s_2	3	9	6	0	1	30
Z	-10	-15	-20	0	0	0

3. Guide learners in converting the inequalities into equations by introducing slack variables. Facilitate a discussion on why slack variables are necessary and how they help establish the initial basic feasible solution.
4. Display the initial simplex tableau and model how to i identify the entering and leaving variables. Ask probing questions that help learners justify their choices using the most negative entry in the objective row and the minimum-ratio test.
5. Lead learners through each pivoting operation. As learners perform row operations, encourage them to explain how the pivot row is normalized and how the remaining entries in the pivot column are eliminated. Monitor their work and address misconceptions related to tableau manipulation.
6. Facilitate a whole-class discussion of the final tableau. Guide learners in interpreting the values of the basic and non-basic variables, determining the maximum value of the objective function, and verifying that the solution satisfies all original constraints. Conclude by discussing how the Simplex Method provides an efficient way to solve optimization problems involving multiple variables and constraints.

Process Questions:
1. Which row shows the objective function?
2. What does the final value of Z represent?
3. Why is the solution considered optimal?

Step 4. Identify the entering variable. The most negative Z-row entry is -20 under x_3, so x_3 enters the basis.

Step 5. Identify the leaving variable. Apply the minimum-ratio test to the x_3 column: 24/6 = 4 and 30/6 = 5. The smallest positive ratio is 4, so s_1 leaves the basis. The pivot element is 6.

Step 6. Normalize the pivot row by dividing the s_1 row by 6. The new x_3 row is [1/3, 2/3, 1, 1/6, 0 | 4].

Step 7. Eliminate the other entries in the x_3 column. R2_new = R2 - 6·R1_new. Z_new = Z + 20·R1_new.

Tableau after iteration 1

Basis	x_1	x_2	x_3	s_1	s_2	RHS
x_1	1/2	2/3	1	1/6	0	4
s_2	1	5	0	-1	1	6
Z	-10/3	-5/3	0	10/3	0	80

Step 8. Repeat the entering-variable test. The most negative Z-row entry is -10/3 under x_1, so x_1 enters the basis.

Step 9. Identify the leaving variable. Apply the minimum-ratio test to the x_1 column: for the x_3 row, 4 / (1/3) = 12; for the s_2 row, 6 / 1 = 6. The smallest positive ratio is 6, so s_2 leaves the basis. The pivot element is 1.

Step 10. Pivot and eliminate the other x1-column entries. The s_2 row already has pivot value 1, so it becomes the new x_1 row. R1_new = R1 - (1/3)·R2_new. Z_new = Z + (10/3)·R2_new.

Final tableau:

Basis	x_1	x_2	x_3	s_1	s_2	RHS
x_3	0	-1	1	1/2	-1/3	2
x_1	1	5	0	-1	1	6
Z	0	15	0	0	10	100

Step 11. Check optimality and read the solution. All Z-row entries are non-negative, so the solution is optimal. Basic variables: $x_1 = 6$ and $x_3 = 2$. Non-basic variable: $x_2 = 0$. Maximum value: Z = 100.

<table>
<tr><td></td><td>Verification against the original constraints:
2(6) + 4(0) + 6(2) = 12 + 0 + 12 = 24 (binding)
3(6) + 9(0) + 6(2) = 18 + 0 + 12 = 30 (binding)
Z = 10(6) + 15(0) + 20(2) = 60 + 0 + 40 = 100</td><td></td></tr>
<tr><td></td><td colspan="2">B.3. Developing Mastery (For the complete instructions for the learners, please refer to the LAS.)</td></tr>
<tr><td></td><td colspan="2">Activity B.3. PRACTICING LEARNED SKILLS

Before facilitating this activity, ensure that all learners have a copy of the LAS.

Demonstrate the initial setup of problem 1’s simplex tableau on the board. Have learners complete problem 1 in pairs while you walk the room. Quickly verify the optimum, then assign problem 2.</td></tr>
<tr><td>C. Demonstrating Knowledge and Skills</td><td colspan="2">C.1. Finding Practical Application</td></tr>
<tr><td></td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

Instructions:
1. Read and analyze the problem. Identify the variables, objective function, and constraints based on the available labor hours and fertilizer units.
2. Write the mathematical model, including the profit function and all constraints.
3. Graph the constraints on a coordinate plane and determine the feasible region.
4. Identify the corner points of the feasible region and evaluate the profit at each point.
5. Determine the combination of tomatoes and eggplants that maximizes profit. Explain and justify the solution using the graph and computations.

SMART FARMING FOR MAXIMUM PROFIT
Mang Roberto supplies vegetables to a local market and a small grocery. He grows tomatoes and eggplants and wants to maximize his profit.

Each crop requires different resources and provides different profits. Tomatoes earn a profit of ₱500 per crate, while eggplants earn ₱400 per crate.</td><td>Activity C.1. MAKING REAL-WORLD CONNECTIONS

1. Present the situation and ask learners to identify the decision that Mang Roberto needs to make. Guide them in defining the variables, objective function, and constraints.
2. Facilitate a discussion on how the information in the problem can be translated into mathematical expressions.
3. Check learners’ formulations and provide support when needed.
4. Instruct learners to graph the constraints and identify the feasible region. Move around the room to monitor progress and clarify misconceptions about graphing and interpreting constraints.
6. Guide learners in determining the corner points of the feasible region and evaluating the profit function at each point. Encourage them to explain their computations and reasoning.
8. Lead a synthesis discussion by asking learners to justify the optimal solution and explain how linear programming can help make informed decisions in real-life situations such as farm management and resource allocation

Processing Questions
1. How did the labor and fertilizer constraints affect the number of crates of tomatoes and eggplants that could be</td></tr>
</table>

Mang Roberto has 120 hours of labor per week. Each crate of tomatoes requires 5 hours of labor, and each crate of eggplants requires 4 hours of labor.

He also has 90 units of fertilizer available weekly. Each crate of tomatoes and eggplants requires 2 units of fertilizer.

How many crates of tomatoes and eggplants should Mang Roberto produce to maximize his profit?

produced?
2. Why is it important to evaluate the profit function at all corner points of the feasible region before selecting the optimal solution?
3. How can linear programming help farmers and business owners make better decisions when resources are limited?

C.2. Making Generalization

Activity C.2. WRAPPING UP THE LESSON

Instructions:
Complete the organizer below to summarize the key concepts learned in solving linear programming problems using the simplex method.

Questions	Responses
How can we determine the best possible solution when there are multiple constraints?	
Why is it important to optimize resources in real-life situations?	
How does the simplex method improve decision-making compared to graphing methods?	
What does an "optimal solution" mean and how do we reach it?	

Activity C.2. WRAPPING UP THE LESSON

1. Distribute or display the organizer and ask learners to r review the activities completed during the lesson on linear programming and the simplex method.
2. Guide learners in reflecting on their learning by discussing each question briefly before they write their responses. Encourage them to connect the concepts to the activities they have completed.
4. Provide sufficient time for individual completion of the organizer. Move around the room to monitor progress and clarify misconceptions about optimization, constraints, and the simplex method.
5. Facilitate a sharing session by inviting selected learners or groups to present their responses. Encourage learners to compare ideas and provide mathematical justifications for their answers.
6. Synthesize the lesson by highlighting how the simplex method identifies optimal solutions efficiently and supports decision-making in situations involving limited resources and multiple constraints.

C.3. Evaluating Learning (For the complete instructions for the learners, please refer to the LAS.)

Activity C.3. ASSESSING LEARNING OUTCOMES

Before facilitating this activity, ensure that all learners have a copy of the LAS
1. To assess learners' understanding of the lesson, facilitate the evaluation activity. After reviewing their responses, guide learners in a brief reflection to evaluate their learning and identify areas for further improvement.
2. Invite selected learners to share their responses to encourage reflection and discussion. Learners' answers are used as

	formative assessment to identify misconceptions and guide remediation or enrichment activities. a. How has your understanding of decision-making changed after learning LP, especially in situations with limited resources? b. Which concepts or skills in Linear Programming did you find easiest or most difficult, and why? c. What aspects of Linear Programming do you still find confusing or need more practice with?.
	C.4. Additional Activities (For the complete instructions for the learners, please refer to the LAS.)
	Activity C.4. EXTENDING AND REINFORCING LEARNING Before facilitating this activity, ensure that all learners have a copy of the LAS Based on the results of the evaluation, facilitate the appropriate learning activities. Learners who have not yet attained mastery should complete the remediation activity, while those who have attained mastery should be provided with the enhancement activity. During the remediation activity, teacher support is essential. Provide timely guidance, feedback, and scaffolding to help learners address misconceptions, strengthen their understanding of the concepts, and progress toward mastery.
V. ASSESSMENT	**To evaluate learners' success in attaining the intended learning competencies, the assessment tools and strategies provided on the link in the table of contents can be utilized to measure understanding, skills, and application of concepts**
VI. REFLECTION	**To assess and evaluate the effectiveness of the instruction, as well as to identify challenges and plan for improvements in this unit, teachers are encouraged to answer the reflective questions provided in the link indicated in the table of contents**